DIFFERENTIABLE AND NON-DIFFERENTIABLE CELLULAR NEURAL NETWORKS WITH IMPLICATIONS IN THE BACTERIAL GROWTH PROCESS. A MATHEMATICAL MODEL (I)

BY

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Abstract. In the framework of the Extended Scale Relativity Theory, the Schrödinger type geodesics and non-differentiable hydrodynamic model are given. In the one-dimensional case of the non-differentiable hydrodynamic model, an analytical solution is obtained in the form of cnoidal oscillation modes. Associating a Toda lattice to the cnoidal oscillation modes, two types of cellular neural networks result by mapping: a differentiable cellular neural network and a non-differentiable cellular neural network. This model can be applied to bacterial growth process.

Keywords: differentiable cellular neural network; non-differentiable cellular neural network; coherence; Toda lattice; cnoidal oscillation modes; soliton; wave; neuron.

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1. Introduction

Many phenomena with complex patterns and structures are widely observed in the brain. These phenomena are some manifestations of a multidisciplinary paradigm called emergence or complexity. They share a common unifying principle of dynamic arrays, namely, interconnections of a sufficiently large number of simple dynamic units can exibit extremely complex and self-organizing behaviors (Karayiannis & Venetsanopoulos, 1993; Chow, 2007).

Standard theoretical models of complex systems dynamics are sophisticated and ambiguous (Flake, 1998; Mitchell, 2009). However, such situation can be simplified if we consider that complexities in interaction processes impose various time resolution scales and the evolution pattern leads to different freedom degrees. To develop new theoretical models, we have to admit that the complex system entities with chaotic behaviours can achieve self-similarity (space-time structures can appear) associated with strong fluctuations at all possible space-time scales. Then, for time scales that prove to be larger if compared with the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaced by a collection of potential routes. In its turn, the concept of "definite positions" is peplaced by that of probability density.

Thus, non-differentiability appears as a universal property of complex systems and, moreover, it is necessary to create non-diffentiability physics of complex systems. Under such circumstances, if we consider that the complexity of interactions in the dynamics of complex systems is replaced by non-differentiability, it is no longer necessary to use the whole classical arsenal of quantities from standard physics (differentiable physics). This topic was developed using either the Scale Relativity Theory (SRT) (Nottale, 2011) and Extended Scale Relativity Theory (ESRT) *i.e.*, SRT with arbitrary constant fractal dimension. On the fundaments of such model and some of its implications, the reader can refer to Gottlieb *et al*., 2006; Gurlui *et al*., 2006, Timofte *et al*., 2011. According to these models, the dynamics of complex system quasi-particles takes place on continuous but non-differentiable curves (fractal curves) so that all physical phenomena involved depend not only on space-time coordinates but also on space-time scale resolution. That is why physical quantities describing the dynamics of complex systems can be considered as fractal functions (Nottale, 2011; Mandelbrot, 1983). Moreover, according to geodesics in a non-differentiable (fractal) space, the complex system quasi-particles may be reduced to, and identified with, their own trajectories (*i.e.*, their geodesics) so that the complex system should behave as a special "fluid" lacking interactions - fractal fluid (Nottale, 2011; Timofte *et al*., 2011).

In the present paper, we consider that the nervous influxe (through brain's neuronal network) takes place on continuous but non-differentiable curves in the hydrodynamic variant of scale relativity (with arbitrary constant fractal dimension). Dynamics types that are compatible with such frame of a

cellular neural network are analyzed. In a such context some implication of the mathematical model in the bacterial growth process are achived.

2. Results and Discussions

2.1. Brief Remainder of the Theory

We can simplify the dynamics in complex systems assuming that its entities (structural units) (Mitchell, 2009) move on continuous but non-differentiable curves. Once accepted such a hypothesis, the dynamics of the entities is given by the non-differentiable operator $\hat{d}/dt$ (Timofte *et al.*, 2011).

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \hat{V} \cdot \nabla - iD(dt)^{(2/D_F)-1}\Delta \tag{1}$$

where

$$\hat{V} = V_D - iV_F \tag{2}$$

is the complex velocity, $\mathbf{V}_D$ is the differentiable and scale resolution independent velocity, $\mathbf{V}_F$ is the non-differentiable and resolution scale dependent velocity, $\hat{V} \cdot \nabla$ is the convective term, $D(dt)^{(2/D_F)-1}\Delta$ is the dissipative term, ∇ is the nabla operator, Δ is the Laplace operator, D_F is the fractal dimension of the motion curve, dt is the scale resolution and D is a specific coefficient associated to the differentiable-non-differentiable transition. For D_F any definition can be used (the Hausdorff-Besikovici fractal dimension, the Kolmogorov fractal dimension etc. (Mandelbrot, 1983)), but once accepted such a definition for D_F, it has to be maintained over the entire analysis of the complex system dynamics. Applying the non-differentiable operator (1) to the complex speed (2) and accepting the scale covariance principle in the form:

$$\frac{\hat{d}\hat{V}}{dt} = -\nabla U \tag{3}$$

we obtain the motion equation

$$\frac{\hat{d}\hat{V}}{dt} = \frac{\partial \hat{V}}{\partial t} + \left(\hat{V} \cdot \nabla\right)\hat{V} - iD(dt)^{(2/D_F)-1}\Delta \hat{V} = -\nabla U \tag{4}$$

where U is the external scalar potential. It means that at any point of a non-differentiable path, the local acceleration term, $\partial_t \hat{V}$, the non-linearly

(convective) term, $(\hat{V} \cdot \nabla)\hat{V}$ the dissipative term, $D(dt)^{(2/D_F)-1}\Delta\hat{V}$ and external force, ∇U makes their balance. Therefore, the complex structure dynamics can be assimilated with the dynamics of a "rheological" fluid. This dynamics is described by the complex velocities field, $\hat{V}$, the complex acceleration field, $\hat{d}\hat{V}/dt$ etc. and by the imaginary viscosity type coefficient, $iD(dt)^{(2/D_F)-1}$.

For irrotational motions of the complex system entities:

$$\nabla \times \hat{V} = 0, \nabla \times V_D = 0, \nabla \times V_F = 0 \tag{5}$$

we can choose $\hat{V}$ of the form

$$\hat{V} = -2iD\left(dt\right)^{(2/D_F)-1}\nabla\ln\Psi \tag{6}$$

where $\varphi \equiv \ln\Psi$ is the velocity scalar potential. By substituting (6) in (4) and using the method described in (Timofte *et al.*, 2011), it results:

$$\frac{d\hat{V}}{dt} = -2iD\left(dt\right)^{(2/D_F)-1}\nabla\left[\frac{\partial\ln\Psi}{\partial t} - 2iD\left(dt\right)^{(2/D_F)-1}\frac{\nabla\Psi}{\Psi}\right] = -\nabla U \tag{7}$$

This equation can be integrated and yields:

$$D^2\left(dt\right)^{(4/D_F)-2}\Delta\Psi + iD\left(dt\right)^{(2/D_F)-1}\frac{\partial\Psi}{\partial t} - \frac{U}{2}\Psi = 0 \tag{8}$$

up to an arbitrary phase vector which may be set to zero by a suitable choice of the phase of Ψ. For motions of complex system entities on Peano's curves, $D_F = 2$, the eq. (8) takes the Nottale's form:

$$D_N^2\Delta\Psi + iD_N\frac{\partial\Psi}{\partial t} - \frac{U}{2}\Psi = 0, \ D \equiv D_N \tag{9}$$

Moreover, for motions of complex systems on Peano curves, $D_F = 2$, at Compton scale, $D = \hbar/2m_0$, with $\hbar$, the reduced Planck constant and m_0 the rest mass of the complex system entities, the relation (8) becomes the standard Schrödinger equation (Phillips, 2003):

$$\frac{\hbar^2}{2m_0}\Delta\Psi + i\hbar\frac{\partial\Psi}{\partial t} - U\Psi = 0 \tag{10}$$

If $\Psi = \sqrt{\rho}\,e^{iS}$, with $\sqrt{\rho}$ the amplitude and S the phase of Ψ, the complex velocity field (6) takes the form

$$\hat{V} = 2D\left(dt\right)^{(2/D_F)-1}\nabla S - iD\left(dt\right)^{(2/D_F)-1}\nabla\ln\rho \tag{11}$$

$$V_D = 2D\left(dt\right)^{(2/D_F)-1}\nabla S \tag{12}$$

$$V_F = D\left(dt\right)^{(2/D_F)-1}\nabla\ln\rho \tag{13}$$

By substituting (11)−(13) in (4) and separating the real and the imaginary parts, up to an arbitrary phase factor which may be set at zero by a suitable choice of the phase of Ψ, we obtain:

$$\frac{\partial V_D}{\partial t} + \left(V_D\cdot\nabla\right)V_D = -\nabla\left(Q+U\right) \tag{14}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho V_D\right) = 0 \tag{15}$$

with Q the specific non-differentiable potential

$$Q = -2D^2\left(dt\right)^{(4/D_F)-2}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{V_F^2}{2} - D\left(dt\right)^{(2/D_F)-1}\nabla\cdot V_F \tag{16}$$

The eq. (14) represents the specific momentum conservation law, while eq. (15) represents the states density conservation law. The eqs. (14)−(16) define the non-differentiable hydrodynamics model (NDHM).

The following conclusions are obvious:

i) Any entity of the complex system is in a permanent interaction with the non-differentiable medium through the specific non-differentiable potential (16);

ii) The non-differentiable medium is identified with a non-differentiable fluid described by the specific momentum and states density conservation laws;

iii) The non-differentiable speed $\mathbf{V}_F$ does not represent actual mechanical motion, but it contributes to the transfer of specific momentum and the concentration of energy. This may be seen clearly from the absence of $\mathbf{V}_F$ from the states density conservation law (15) and from its role in the variational principle (Nottale, 2011);

iv) Any interpretation of Q should take cognizance of the "self" nature of the specific momentum transfer. While the energy is stored in the form of the mass motion and potential energy (as clasically is), some is available elsewhere

and only the total is conserved. It is the conservation of the energy and specific momentum that ensure reversibility and the existence of eigenstates, but denies a Brownian motion of interaction with an external medium (Nottale, 2011);

v) Since the position vector of the complex system entity is assimilated to a stochastic process of Wiener type (Mandelbrot, 1983), Ψ is not only the scalar potential of complex speed (through $\ln\Psi$) in the frame of non-differentiable hydrodynamics, but also density of probability (through $|\Psi|^2$) in the frame of a Schrödinger type theory. It results the equivalence between the formalism of the non-differentiable hydrodynamics and the ones of the Schrödinger type equation. Moreover, the chaoticity, either through turbulence in the non-differentiable hydrodynamics approach or through stochasticization in the Schrödinger type approach, is generated by the non-differentiability of the movement trajectories in a fractal space.

2.2. Cnoidal Modes, Toda Lattices and Cellular Neural Networks

In one dimensional case, the eqs. (14)−(16) in non-dimensional coordinates

$$\omega t = \tau, kx = \xi, \theta = \xi - M\tau$$

$$\frac{V_D}{V_{0D}} = v, \frac{\rho}{\rho_0} = N, \frac{kV_{0D}}{\omega} = 1 \tag{17}$$

for $U = -E\rho$ with $E =$ const., through integration and by eliminating the velocities field, become

$$\frac{k^2 D^2 \left(dt\right)^{(4/D_F)-2}}{V_{0D}^2} \cdot \frac{1}{N^{\frac{1}{2}}} \cdot \frac{d^2 N^{\frac{1}{2}}}{d\theta^2} = \frac{E\rho_0}{V_{0D}} N + \frac{M^2}{2} - \frac{c_1^2}{2N^2} + c_2 \tag{18}$$

In the above relations ω is a critical pusation, k is the inverse of a critical length, V_{0D} is a critical velocity, ρ_0 is a critical density, M is an equivalent of Mach number and c_1, c_2 are two integration constants.

Using the standard procedure (Timofte *et al.*, 2011), one obtains the solution

$$N = \bar{N} + 2a\left[\frac{E(s)}{K(s)} - 1\right] + 2a \cdot cn^2\left[\frac{\sqrt{a}}{s}(\theta - \theta_0); s\right] \tag{19}$$

In relations (19), $\overline{N}$ is the average value of the normalized states density, a is the amplitude, s is the modulus of the complete elliptical

integrals of the first and second kind, $K(s)$ and respectively $E(s)$, and *cn* is the Jacobi elliptical function of modulus s (Armitage, 2006) and argument $\sqrt{a}(\theta - \theta_0)/s$. The modulus s is a control parameter of non-linearity, depending on both the intrinsic properties of the complex system and of the dynamics regimes. Therefore, the dynamics of the complex system is achieved through space-time cnoidal oscillation modes of the normalized states density. The cnoidal oscillation modes are associated to the Toda lattice (Toda, 1981).

Let us note that the elliptic function *cn* is double periodical, the real period being dependent of the modulus s, while the imaginary one is dependent of the complementary modulus s' with conditioning $s^2 + s'^2 \equiv 1$. Then, by (19), two solutions are actually explained, one of them being specific to the states density at differentiable scale induced by the modulus s and the other one being specific to the states density at non-differentiable scale induced by the complementary modulus s'. To these two solutions are now associated two Toda type sub-networks Toda, one of them being specific to the differentiable scale (which will be called *differentiable Toda lattice* (DTL)) and the other one being specific to the non-differentiable scale (which will be called *non-differentiable Toda lattice* (NDTL)). Accordingly (Jackson, 1992; Chua & Yang, 1988a; Chua & Yang, 1988b) by mapping there result two cellular neural networks, one of them being specific to the differentiable scale (we shall call it *differentiable cellular neural network* (DNNC)) and the other one being specific to the non-diferentiabile scale (will be called *non-differentiable cellular neural network* (NDNNC)). These two networks, even they have distinct functionalities, one of them being associated to "the outer" of the complex system through the diferentiable scale and the other one being associated to the "inner" of the same complex system through the non-diferentiable scale, simultaneously coexist and are interdependent. Practically, together with the differentiable-non-differentiable Toda lattice pair, the differentiable-non-differentiable cellular neural network pair is generated by mapping.

The self similarity of the space-time cnoidal oscillation modes specifies the existence of some "cloning" mechanisms (full and fractional wave function revivals - a wave functions evolves in time to a state describable as a collection of spatially distributed sub-wave functions that each closely reproduces the initial wave function shape (Aronstein & Stroud, 1997)). All these show a direct connection between the non-differentiable structure of the "dynamics regimes" and holographic principle (in any finite volume of a complex system it finds its entire structure) (Mandelbrot, 1983; Nottale, 2011).

In our opinion the previous mechanisms of the human brain play a fundamental role in the bacterial growth process (under external constraints such as injuries the adequate "software" in the brain comes into motion, thus activating specific bacteria. We note that "Bacterial growth is the asexual

reproduction or <u>cell division</u>, of a <u>bacterium</u> into two daughter cells, in a process called <u>binary fission</u>. Providing no mutational event occurs the resulting daughter cells are genetically identical to the original cell. Hence local doubling of the bacterial population occurs. Both daughter cells from the division do not necessarily survive. However, if the number surviving exceeds unity on average, the bacterial population undergoes exponential growth. The measurement of an exponential bacterial growth curve in batch culture was traditionally a part of the training of all microbiologists; the basic means requires bacterial enumeration (cell counting) by direct and individual (microscopic, flow cytometry), direct and bulk (biomass), indirect and individual (colony counting), or indirect and bulk (most probable number, <u>turbidity</u>, nutrient uptake) methods. Models reconcile theory with the measurements" (Skarstad *et al*. 1983; Zwietering *et al*. 1990; https://en.wikipedia.org/wiki/Bacterial_growth#cite_notepmid6341358-1).

3. Conclusions

The main conclusions of the present paper are the following:

i) In the ESRT, the Schrödinger type geodesics and non-differentiable hydrodynamic equations are obtained;

ii) In the one-dimensional case, a solution of the non-differentiable hydrodynamic equations in the form of cnoidal oscillation modes is given;

iii) Associating a one-dimensional Toda lattice to the cnoidal oscillation modes, two cellular neural networks result by mapping, one of them being specific to the differentiable scale (differentiable cellular neural network (DNNC)) and the other one being specific to the non-differentiable scale (non-differentiable cellular neural network (NDNNC));

iv) Specific mechanisms required by the cellular neural network of the brain in the bacterial growth process are analyzed.

Implications of these cellular neural networks in the functionality of some biological systems are given in Duceac *et al*., 2015a and Duceac *et al*., 2015b.

REFERENCES

Armitage J.V., *Elliptic Functions*, Cambridge University Press, Cambridge, 2006.
Aronstein D.R., Stroud C.L., *Fractional Wave-Function Revivals in the Infinite Square Well*, Physical Review A, **55**, *6*, 4526−4537, 1997.
Chow T.W.S., *Neural Networks and Computing,* Learning Algorithms and Applications, Series in Electrical and Computer Engineering, 2007.
Chua L.O., Yang L., *Cellular Neural Networks: Applications,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1273−1290, 1988a.
Chua L.O., Yang L., *Cellular Neural Networks: Theory,* IEEE Transactions on Circuits and Systems, **35**, *10*, 1257−1272, 1988b.

Duceac L.D., Nica I., Gaţu I., *Fractal Bacterial Growth,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 57−64, 2015a.

Duceac L.D., Nica I., Iacob D.D., *Chaos and Self-Structuring in Cellular Growth Dynamics,* Bul. Inst. Polit. Iaşi, **LXI (LXV)**, *1*, 65−72, 2015b.

Flake G.W., *The Computational Beauty of Nature,* MIT Press, Cambridge, MA, 1998.

Gottlieb I., Agop M., Jarcău M., *El Naschie's Cantorian Space-Time and General Relativity by Means of Barbilian's Group. A Cantorian Fractal Axiomatic Model of Space-Time,* Chaos, Solitons and Fractals, **19**, *4*, 705−730, 2004.

Gurlui S., Agop M., Strat M., Băcăiţă S., Cerepaniuc A., *Some Experimental and Theoretical Results on the on the Anodic Patterns in Plasma Discharge,* Physics of Plasmas, **13**, I, 2006.

https://en.wikipedia.org/wiki/Bacterial_growth#cite_notepmid6341358-1

Jackson E.A., *Perspectives on Nonlinear Dynamics,* Cambridge University Press, Cambridge, 1992.

Karayiannis N.B., Venetsanopoulos A.N., *Artificial Neural Networks. Learning Algorithms, Performance Evaluation, and Applications,* The Springer International Series in Engineering and Computer Science, **209**, 1993.

Mandelbrot B., *The Fractal Geometry of Nature,* Updated and Augm. Ed., W.H. Freeman, New York, 1983.

Mitchell M., *Complexity: A Guided Tour,* Oxford University Press, Oxford, 2009.

Nottale L., *Scale Relativity and Fractal Space -Time - A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London, 2011.

Phillips A.C., *Introduction to Quantum Mechanics,* Wiley, New York, 2003.

Skarstad K., Steen H.B., Boye E., *Cell Cycle Parameters of Slowly Growing Escherichia Coli B/r Studied by Flow Cytometry,* J. Bacteriol., ***154***, *2*, 656−662, PMC 217513, PMID 6341358 (1983).

Timofte A., Casian-Botez I., Scurtu D., Agop M., *System Dynamics Control Through the Fractal Potential,* Acta Physica Polonica A, **119**, *3*, 304−311 (2011).

Toda M., *Theory of Nonlinear Lattices,* New York: Springer Verlag, 1981.

Zwietering M.H., Jongenburger I., Rombouts F.M., van 'T Riet K., *Modeling of the Bacterial Growth Curve,* Applied and Environmental Microbiology, **56**, *6*, 1875−1881, PMC 184525, PMID 16348228 (1990).

RETELE NEURALE CELULARE DIFERENŢIABILE ŞI
NEDIFERENŢIABILE CU IMPLICAŢII ÎN PROCESUL DE CREŞTERE A
BACTERIILOR. UN MODEL MATEMATIC (I)

(Rezumat)

În cadrul Teoriei Extinse a Relativităţii de Scală, sunt obţinute geodezicele de tip Schrödinger şi cele ale modelului hidrodinamic. În cazul unidimensional al modelului hidrodinamic nediferenţiabil, este obţinută o soluţie analitică sub forma modurilor de oscilaţie cnoidale. Asociind modurilor de oscilaţii cnoidale o reţea de tip Toda, prin mapare, rezultă două tipuri de reţele celulare neurale, o reţea celulară neurală diferenţiabilă şi o reţea celulară neurală nediferenţiabilă. Câteva implicaţii ale modelului în mecanismul de creştere al bacteriilor este de asemenea considerat în cadrul lucrării.