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**DIFFERENTIABLE AND NON-DIFFERENTIABLE CELLULAR  
NEURAL NETWORKS WITH IMPLICATIONS IN THE  
BACTERIAL GROWTH PROCESS. PROPERTIES (II)**

BY

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**Abstract.** Properties of cellular neural networks (differentiable and non-differentiable cellular neural networks) are given: wave number spectrum, phase velocity spectrum, quasi-period spectrum etc. In such perspective, the conditions of non-differentiable-non-differentiable cellular neural networks coherence are given and this could explain not only the way in which information is stored and transmitted in the brain but also the way in which the communications codes can be generated. Some implications of the model in the growth bacterial process are established.

**Keywords:** differentiable cellular neural network; non-differentiable cellular neural network; coherence; information; communication codes.

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## 1. Introduction

According to Gavriluț *et al.*, 2016, two types of cellular neural networks (differentiable and non-differentiable cellular neural networks) are obtained. These cellular neural networks have some specific properties that will be highlighted in this paper.

## 2. Results and Discussions

### 2.1. Specific Properties of the Cellular Neural Networks

The Toda lattices at differentiable and non-differentiable scales and by mapping the two corresponding associated cellular networks have some following characteristic parameters:

i) Wave number spectrum

$$k(s) = \frac{\pi a^{1/2}}{sK(s)}, k'(s') = \frac{\pi a^{1/2}}{s'K(s')} \quad (1)$$

ii) Phase velocity spectrum

$$U_p(s) = 6\bar{N} + 4a \left[ \frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2} \right], U'_p(s') = 6\bar{N} + 4a \left[ \frac{3E(s')}{K(s')} - \frac{1+s'^2}{s'^2} \right] \quad (2)$$

iii) Quasi-period spectrum

$$T(s) = \frac{1}{\frac{3\pi\bar{N}a^{1/2}}{sK(s)} + \frac{2a^{3/2}}{sK(s)} \left[ \frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2} \right]}$$

$$T(s') = \frac{1}{\frac{3\pi\bar{N}a^{1/2}}{s'K(s')} + \frac{2a^{3/2}}{s'K(s')} \left[ \frac{3E(s')}{K(s')} - \frac{1+s'^2}{s'^2} \right]} \quad (3)$$

The significances of the quantities from (1)–(3) are given in Gavriluț *et al.*, 2016.

Expliciting the oscillation modulus of the Toda network and the functionality of the associated cellular neural network at differentiable scale is achieved by the following degenerations of the elliptic function *cn* in the modulus *s* (Armitage, 2006).

i) For  $s \rightarrow 0$ , the solution (19) from the paper of Gavriluț *et al.*, 2016 reduces to the harmonic package type sequence:

$$N = \langle N \rangle + a \cos(\bar{\theta}), \quad \langle N \rangle = \bar{N} + a \quad (4)$$

which is described by the wave number spectrum

$$k \cong \frac{2a^{1/2}}{s} \quad (5)$$

the phase velocity spectrum:

$$U_p \cong 6\bar{N} + 8a - k^2 \quad (6)$$

and the quasi-period spectrum

$$T \cong \frac{2\pi}{6\bar{N}k + 8ak - k^3} \quad (7)$$

ii) For  $s \rightarrow 1$ , the solution (19) from Gavriluț *et al.*, 2016 reduces to a soliton package type sequence:

$$N \cong \bar{N} + a_1 \operatorname{sech}^2 \left[ \left( \frac{a_1}{6} \right)^{1/2} (\theta - \theta_0) \right] \quad (8)$$

characterized by the wave number spectrum

$$\Lambda \cong \frac{(2a_1)^{1/2}}{4k_1}, a_1 = 2a, k_1 = \frac{k}{2\pi} \quad (9)$$

the phase velocity spectrum

$$u_p \cong 6\bar{N} + 2a_1 - 12k_1^2 a_1^{1/2} \quad (10)$$

and the quasi-period spectrum

$$T \cong \frac{1}{6\bar{N}k_1 + 2a_1k_1 - 12k_1^2 a_1^{1/2}} \quad (11)$$

For  $s = 0$ , the solution (19) from Gavriluț *et al.*, 2016 reduces to an harmonic type sequence, while for  $s = 1$  it reduces to a soliton type one. For details on the degeneration of elliptic function  $cn$  see Armitage, 2006, while for details related to nonlinear solutions (soliton, soliton package etc.) (Jackson, 1992).

As shown before, nonlinear waves have long been an interest to scientists in a variety of disciplines. A multitude of applications have been employed, ranging from fluid dynamics and plasma physics to even neuroscience and biology. It is with this same interest that we have chosen the topic of computational waves. Waves are omnipresent, from tsunamis in the ocean, to gamma waves and sonic booms. In fact, there are currently several wave pulses propagating through the neurons in our brain due to the firing of action potentials.

In the physics literature, the terms soliton and solitary wave are often used interchangeably. Solitary waves (and solitons) arise in both continuous systems such as the KdV equation and discrete systems such as the Toda lattice (Toda, 1981; Toda, 1983) and in both one and multiple spatial dimensions. Key issues in studying solitary waves also include linear versus nonlinear (of course), integrable versus nonintegrable, persistent versus transient, asymptotics (*i.e.*, consideration of time scales), localization in physical space versus Fourier space, and the effects of noise.

The important property of the Toda equation is the existence of so called soliton solutions, that is, pulslke waves which spread in time without changing their size or shape and interact with each other in a particle-like way. This is a surprising phenomenon, since for a generic linear equation one would expect spreading of waves (dispersion) and for a generic nonlinear force one would expect that solutions only exist for a finite time (breaking of waves). Obviously, our particular force is such that both phenomena cancel each other giving rise to a stable wave existing for all time (Jackson, 1992).

Simultaneously, it is also achieved the explanation at non-differentiable scale of the Toda network oscillation modes and of the associated cellular neural network functionalities through the elliptic functions  $cn$  degenerations in the complementary modulus  $s'$ .

Eliminating the amplitude  $a$  between (1) and (2) we obtain the following dispersion equations:

$$\begin{aligned} (U_p - 6\bar{N})\bar{\lambda}^2 &= 16A(s), k = \frac{2\pi}{\lambda}, \\ (U'_p - 6\bar{N})\bar{\lambda}'^2 &= 16A'(s'), k' = \frac{2}{\lambda'} \end{aligned} \quad (12)$$

where

$$\begin{aligned} A(s) &= 3s^2 K(s)E(s) - (1+s^2)K^2(s) \\ A'(s') &= 3s'^2 K(s')E(s') - (1+s'^2)K'^2(s') \end{aligned} \quad (13)$$

Nonlinearity  $s$  generates two distinct dynamics regimes of the Toda lattice respectively of the cellular neural network associated to the differentiable scale: non-quasi-autonomous regimes (by harmonic type sequences, harmonic package type sequences) and quasi-autonomous regimes (by soliton type sequences, soliton package type sequences) respectively. The dependency  $A(s)$  - see Fig. 2 a,b, specifies that the value  $s \cong 0.7$  separates the two dynamic regimes. For  $0 \leq s \leq 0.7$ , *i.e.*, in non-quasi-autonomous regime, the variable  $A(s) \cong \text{const.}$ , situation in which the first of eqs. (12) takes the form:

$$(U_p - 6\bar{N})\lambda^2 \cong \text{const.} \quad (14)$$

while for  $0.7 \leq s \leq 1$ , *i.e.*, in quasi-autonomous regime, the relation (14) loses its validity. Simultaneously, the non-linearity  $s'$  generates two distinct dynamics regimes of the Toda lattice respectively of the cellular neural network associated to the non-differentiable scale.

## 2.2. The Coherence of Differentiable-non-Differentiable Cellular Neural Network

According to our previous results, differentiable-non-differentiable Toda lattice pair is mathematically described by the elliptic function  $cn^2$ . Now, invoking a fundamental theorem from elliptic functions theory, it is well known that two elliptic functions are equivalent if and only if there exists a homographic transformation between the ratio of their periods (Armitage, 2006):

$$\mu' = \frac{a\mu + b}{c\mu + d}, a, b, c, d \in N \quad (15)$$

where  $\mu'$  and  $\mu$  define the ratios of the these elliptic functions periods. Moreover, if such theorem is fulfilled, then there exist an algebraic relation between these two elliptic functions.

In such a framework, we shall interpret elliptic function equivalence theorem as a coherence condition of differentiable-non-differentiable Toda lattice pairs and, moreover, its consequence, as a “communication” (“relationship”) way of pairs.

Now, by mapping, the previous results imply that differentiable-non-differentiable cellular neural network pairs are coherent if and only if the condition (15) is fulfilled. Moreover, if this condition is fulfilled, then the pairs can “store” and “transmit” information in a specific algebraic language.

### 3. Conclusions

Assuming that the nervous impulse transmission through brain's neuronal network is achieved on continuous and non-differentiable curves, the hydrodynamic version of scale relativity in arbitrary constant fractal dimension is presented (non-differentiable hydrodynamics) in Gavriluț *et al.*, 2016. In such context, the one-dimensional solution of the non-differentiable hydrodynamic model for states density is obtained in the form of spatio-temporal cnoidal modes, assuming that the external scalar potential is proportional with the states density. Moreover, the spatio-temporal cnoidal modes are assimilated to a one-dimensional Toda network and hence, by mapping, to a cellular neural network.

In the present paper, we give some specific properties of the two types of cellular neural networks. Then the following main conclusions result:

i) Cnoidal modes double periodicity induces differentiable-non-differentiable Toda pair and hence, by mapping, differentiable-non-differentiable cellular neural network pair. Pairs components are simultaneously generated, are interdependent and are characterized by some parameters such as wave number spectrum, phase velocity spectrum and quasi-period spectrum;

ii) Each of pair's components presents two functionality regimes, one of them being induced by the harmonic waves and harmonic wave packages and the other one by solitons and packages of solitons;

iii) The dispersion equations are obtained for each of the two pair's components;

iv) Pairs' coherence imposes "storage" and "transmission" of informations in the form of a specific algebraic language.

Unlike the electronic calculator which has a hard structure divided by artificial algorithms, the spectral component corresponding to the hardware particles, has the same artificial behaviour, without having the fractality of the natural development.

As a result, there is no consistency between the cellular network of the substance and the spectral undulatory one. The development of the neural network is performed according to fractal criteria, the same as for all the other parts and systems of the human body. Consequently, the spectrum field created by the undulation of the particles of the neural network is consistent, allowing the information to be processed inside the neural network and also in the spectrum field (Hilbert space), where there are the aspatial and atemporal components which enable the memory and also the complex component that allows the possibility of multidimensional processing which can explain the superior psychic processes, the conceptualization, the semantics, the abstract etc.

Therefore, at any scale there are the two types of realities that coexist, the differential and non-differential, highlighted by the hydrodynamic theory and the stochastic one. Another major difference between the electronic

calculator and the human brain is done by the analogical feature of psychic processing, unlike the digital signal processing. Analog signal processing is enhanced by the topology configuration character of the processing, being not only a numeric processing but also one determined by the geometric topology.

The dimensional dynamics, from 0 to infinite, which is realized in our reality up to 3 dimensions, can be performed multidimensionally in the complex field of psychic reality (through the fractal potential).

In our opinion the previous mechanisms of the human brain play a fundamental role in the growth bacterial process. Once the cellular neural network is activated, specific communication codes are also employed by the relevant cells.

Some implications of cellular neural networks properties in the functionality of biological systems are given also in the references (Duceac *et al.*, 2015a; Duceac *et al.*, 2015b).

Finally, we give an example of how the brain controls, by means of specific softwares, the bacterial growth processes. “The genus *Mycobacterium* is best known for its two major pathogenic species, *M. tuberculosis* and *M. leprae*, the causative agents of two of the world's oldest diseases, tuberculosis and leprosy, respectively. *M. tuberculosis* kills approximately two million people each year and is thought to latently infect one-third of the world's population. One of the most remarkable features of the nonsporulating *M. tuberculosis* is its ability to remain dormant within an individual for decades before reactivating into active tuberculosis. Thus, control of cell division is a critical part of the disease. The mycobacterial cell wall has unique characteristics and is impermeable to a number of compounds, a feature in part responsible for inherent resistance to numerous drugs. The complexity of the cell wall represents a challenge to the organism, requiring specialized mechanisms to allow cell division to occur. Besides these mycobacterial specializations, all bacteria face some common challenges when they divide. First, they must maintain their normal architecture during and after cell division. In the case of mycobacteria, that means synthesizing the many layers of complex cell wall and maintaining their rod shape. Second, they need to coordinate synthesis and breakdown of cell wall components to maintain integrity throughout division. Finally, they need to regulate cell division in response to environmental stimuli” (Hett & Rubin, 2008).

Moreover in the same work (Hett & Rubin, 2008) “discuss these challenges and the mechanisms that mycobacteria employ to meet them. Because these organisms are difficult to study, in many cases we extrapolate from information known for gram-negative bacteria or more closely related GC-rich gram-positive organisms”.

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REȚELE NEURALE CELULARE DIFERENȚIABILE ȘI  
NEDIFERENȚIABILE CU IMPLICAȚII ÎN PROCESUL DE CREȘTERE A  
BACTERIILOR. PROPRIETĂȚI (II)

(Rezumat)

Sunt prezentate proprietăți ale rețelelor celulare neurale (rețele celulare neurale diferențiable și nediferențiable): spectrul numărului de undă, spectrul vitezei de fază, spectrul cvasi-perioadei etc. Din această perspectivă sunt precizate condițiile pentru coerența rețelelor celulare neurale diferențiable și nediferențiable, ceea ce poate explica nu numai modul în care se stochează și transmite informația, ci și modul în care pot fi generate codurile de comunicare. Sunt prezentate câteva exemple prin care codurile de comunicare intervin în procesele de creștere celulară.