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THE ACOUSTIC WORLD: MECHANICAL INERTIA OF WAVES

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Abstract. In this paper, we analyze the phenomena that occur in a medium where the energy and information travels at the speed of mechanical waves. Disturbances are generated, evolves and propagates in the medium correlates with the acoustic waves (mechanical) having the maximum speed. The material environment and the disturbances that occur in it (waves, wave packets, bubbles, etc.) represents the acoustic world. We demonstrate that the wave has the equivalent mass. The equivalent mass of waves is linked to energy by an Einstein type relationship. Events in the acoustic world are linked by Lorentz type transformations. Studying the behavior of standing waves relative to an accelerated observer (Rindler observers) show that the equivalent mass of the wave is the inertial mass. For propagating waves, between linear momentum and generalized momentum (action variable) we find a de Broglie type relationship where the wave momentum is proportional to the wave number and the coefficient of proportionality is the action variable.

Keywords: mechanical waves; Rindler observers; equivalent mass of waves; de Broglie relation.

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1. Introduction

For an electromagnetic universe, *i.e.* a universe those material systems and phenomena are correlated through electromagnetic waves and all events in different reference frames are linked by Lorentz transformations for inertial and by Rindler transformations for non-inertial systems. In the acoustic world, that universe where systems (waves, wave packets, bubbles, etc.) and mechanical phenomena are related by where events are linked by Lorentz type transformations for inertial and non-inertial systems Rindler type transformations. These transformations are obtained from the Lorentz and Rindler transformations replacing the speed of light c with speed of mechanical waves u. In the acoustic world, it was highlighted the existence of acoustic holes like black holes (gravitational and electromagnetic holes) (Unruh, 1995; Weinfurtner *et al*., 2013; Visser, 1998; Barceló *et al*., 2010; Barceló *et al*., 2011; Nandi *et al*., 2004). Observers from acoustic world have senses which are based on the perception of mechanical waves (any form of mechanical perturbation) and instruments that detect and acquire all information through the same type of wave (Nandi *et al*., 2004). Their measurements for time and length (and other parameters) are linked by Lorentz type transformations where the maximum speed is the speed of acoustic waves. Applying these transformations to the of standing waves packet, that energy, mass, linear and generalized momentum undergo relativistic changes.

In the second section, we calculate the energy of the wave that propagates in a fluid and the same calculations for standing wave. We calculate the macroscopic parameters of oscillators having a length equal to one half-wave (tube closed at both ends) or quarter wavelength (tube closed at one end and open at the other). For these oscillators, can define and calculate when generalized momentum (action variable) corresponding to the oscillation energy. The same parameters are calculated based on the constituent particles (atoms, molecules) energy and action.

In the third section, we express the energy as a function of the wave propagation speed. It highlights the existence of an equivalent mass of wave and Einstein type relation between energy and the mass. Also, the linear momentum of the wave (for wave that propagates) is defined. Between this linear momentum and generalized momentum (action variable) exist a de Broglie type relationship that is, the wave's momentum is proportional to the wave number, the coefficient of proportionality is the action variable.

The fourth section we study how to transform energy, mass, linear momentum and action variable of standing waves relative to an inertial observer and relative to a non-inertial observer. We highlight a rest mass and rest energy also a relativistic mass and energy (in inertial motion). Using Rindler type transformations, we show that the macroscopic oscillators equivalent mass is the inertial mass.

The fifth section the conclusions are presented.

2. Mechanical Waves

2.1. The Standing Wave in Gas

Consider a gas found in an tube-form enclosure with section $S = l_x l_y$ and length much greater than the transverse dimensions $l_z >> l_x, l_y$. The gas has the following properties: temperature *T*, pressure p_0, density ρ_0 and mass $m = \rho_0 S_{xy} l_z$. From a microscopic point of view, the gas is constituted of particles (atoms, molecules, etc.) with the mass m_a and density of the particles n_0 so the density is givenby $\rho_0 = n_0 m_a$.

The speed of propagation of mechanical waves in fluid is (Landau and Lifchitz, 1971, Ch. 8):

$$u = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_S} \tag{1}$$

For a gas, the wave propagation is an adiabatic process (constant entropy S) and $(\partial p/\partial \rho)_S$ can be expressed by adiabatic coefficient according to relationship

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \frac{c_p}{c_V}\left(\frac{\partial p}{\partial \rho}\right)_V = \gamma\left(\frac{\partial p}{\partial \rho}\right)_V . \tag{2}$$

For a perfect gas, the speed becomes:

$$u = \sqrt{\frac{\gamma k_B T}{m_a}} . \tag{3}$$

If the tube is closed at both ends, there is created a longitudinal standing wave. The allowed modes are given by the expression of wave number (provided that the length of the rod to be equal to an integer of half-wavelength $l_z = j\lambda/2;\ j = 1,2,3,......$)

$$k_j = j\frac{\pi}{l_z};\ j = 1,2,3,....... \tag{4}$$

From the definition of the wave phase velocity

$$u = \frac{\omega}{k} , \tag{5}$$

results the expression of angular frequency for mode j

$$\omega_j = u k_j = j\frac{\pi u}{l_z} . \tag{6}$$

The quantity

$$\omega_{\min} = \frac{\pi u}{l_z} = \frac{\pi u}{a N_z}. \tag{7}$$

is the minimum angular frequency corresponding mode $j = 1$.

The quantity

$$\omega_{\max} = \frac{\pi u}{l_z} = \frac{\pi u}{a} \tag{8}$$

is the maximum angular frequency corresponding mode $j = N_z$. In relations (7) and (8), the quantity a

$$a = n^{-3}. \tag{9}$$

is the average distance between gas particles at pressure p_0.

To the angular frequencies given by (7) and (8) correspond to the wavelength of maximum

$$\lambda_{\max} = 2 l_z \tag{10}$$

and the minimum wavelength

$$\lambda_{\min} = 2a. \tag{11}$$

For a tube closed at one end, the length of the macroscopic oscillator is $\lambda/4$ because the modes are $l_z = (2j+1)\lambda/4;\ j = 0,1,2,3,...$.

2.2. The Energy of Oscillation of the Standing Wave

For a longitudinal standing wave (Butikov, 2013), each element of length dz and mass $dm = \rho S dz$ executes an oscillatory movement and has kinetic energy ($\Psi_s(z,t) = q_{sz} = q_{0sz}\sin(kz)\sin(\omega t)$)

$$dE_k = \frac{1}{2} dm \left(\frac{\partial q_{sz}}{\partial t} \right)^2 = \frac{1}{2} dm q_{0sz}^2 \omega^2 \sin^2(kz)\cos^2(\omega t) \tag{12a}$$

and the potential energy

$$dE_p = \frac{1}{2} dm u_l^2 \left(\frac{\partial q_{sz}}{\partial z} \right)^2 = \frac{1}{2} dm q_{0sz}^2 \omega^2 \cos^2(kz)\sin^2(\omega t). \tag{12b}$$

In the above relations, q_{sz} is instantaneous amplitude and q_{0sz} is maximum amplitude.

The total energy is

$$dE = dE_k + dE_p = \frac{1}{2} dm q_{0sz}^2 \omega^2 \left[\sin^2(kz)\cos^2(\omega t) + \cos^2(kz)\sin^2(\omega t) \right]. \tag{13}$$

With (13), the oscillation energy of the gas in the tube is

$$E = \frac{1}{2}\rho S q_{0sz}^2 \omega^2 \int_0^{l_z} \left[\sin^2(kz)\cos^2(\omega t) + \cos^2(kz)\sin^2(\omega t) \right] dz = \frac{1}{4}\rho S l_z q_{0sz}^2 \omega^2 = \frac{1}{4} m q_{0sz}^2 \omega^2 . \tag{14}$$

For mode j, with $\lambda_j = 2l_z / j$, the gas energy in the tube in standing wave mode is

$$E_j = \frac{j}{8}\rho l_x l_y q_{0szj}^2 \omega_j^2 \lambda_j = j\left(\frac{\lambda_j}{2} Sn\right)\left(\frac{1}{4} m_a q_{0szj}^2 \omega_j^2\right) = jN_{\lambda_j/2} E_{aj} \tag{15}$$

with $N_{\lambda_j/2}$ the number of microscopic/atomic oscillators from a portion of the tube with length equal to a half-wave

$$N_{\lambda_j/2} = \frac{\lambda_j}{2} Snj \tag{16}$$

and

$$E_{aj} = \frac{1}{4} m_a q_{0szj}^2 \omega_j^2 \tag{17}$$

the energy of a microscopic (atomic) oscillator.

Because the wavelength can be expressed by the speed of propagation u and pulsation ω_j

$$\lambda_j = \frac{2\pi u}{\omega_j}, \tag{18}$$

results, replacing in Eq. (15), the energy of the gas, as a macroscopic oscillator, is proportional to the angular frequency

$$E_j = j\left(\frac{\pi}{4}\rho S q_{0szj}^2 u\right)\omega_j = jJ_{\lambda_j/2}\omega_j . \tag{19}$$

From analytical mechanics (Fasano and Marmi, 2006, p. 431) the energy of the oscillator using harmonic coordinates (the action-angle variables) has the expression (19) where

$$J_{\lambda_j/2} = \left(\frac{\lambda_j}{2} Sn\right)\left(\frac{1}{4} m_a q_{0zj}^2 \omega_j\right) = N_{\lambda_j/2} J_{aj} \tag{20}$$

it is generalized momentum (the canonical action variable) and

$$E_{\lambda_j/2} = \left(\frac{\pi}{4}\rho S q_{0szj}^2 u\right)\omega_j = J_{\lambda_j/2}\omega_j \tag{21}$$

is the macroscopic oscillator energy corresponding to a half-wavelength.

In Eq. (20), the parameter

$$J_{aj} = \frac{1}{4} m_a q_{0zj}^2 \omega_j \tag{22}$$

is the generalized momentum (the canonical action variable) for microscopic (atomic) oscillator energy is given by (17).

For a given material, the oscillator energy and generalized momentum have the lowest value for thermal oscillations in solids, and thermal motion in fluids at temperature $T \neq 0\mathrm{K}$.

The perturbations (the waves and waves packets formed through interference-diffraction) in this environment, an "acoustic world", are correlated with the maximum speed, the speed of waves in the environment. Oscillations produced in this material overlap these perturbations and thermal movements. In this world, the group velocity of the perturbations (the speed with which propagates the amplitudes) is less (or equal) than the speed of waves, $\upsilon = \upsilon_g \leq u$. It follows that the lowest values of generalized $J_{\min}$ momentum and energy $E_{\min}$

$$J_{\min} = J_{aT} = \frac{1}{2} m_a q_{0zT}^2 \omega_T , \tag{23}$$

$$E_{\min} = E_{aT} = \frac{1}{2} m_a q_{0zT}^2 \omega_T^2 \tag{24}$$

correspond to thermal oscillations for microscopic oscillators.

2.3. The Oscillation Energy of the Bar in Presence of a Wave

For a wave with frequency ω and the maximum amplitude q_{0z} , $\Psi(z,t) = q_z = q_{0z} \sin(\omega t - kz)$, which propagates in the gas founded in a tube (the tube has $l_z >> \lambda$), the tube can be considered as a system of oscillators of length $\lambda/2$. Using the same relations (12) and (13) for kinetic, potential and total energy of a half-wave is

$$E_{\lambda/2} = \left(\frac{\pi}{2} \rho S q_{0z}^2 u \right) \omega = J_{\lambda/2} \omega \tag{25}$$

and generalized momentum (the canonical action variable) is

$$J_{\lambda/2} = \frac{\pi}{2} \rho l_x l_y q_{0z}^2 u . \tag{26a}$$

or

$$2\pi J_{\lambda/2} = \pi^2 \rho S q_{0z}^2 u . \tag{26b}$$

The energy and the generalized momentum can be expressed as function of microscopic quantities: mass of atoms m_a and density n

$$E_{\lambda/2} = \left(\frac{\pi}{2} n m_a S q_{0z}^2 u \right) \omega = \left(nS \frac{\lambda}{2} \right) \left(\frac{\pi}{\lambda} m_a q_{0z}^2 u \omega \right) = N_{\lambda/2} E_a \tag{27}$$

$$J_{\lambda/2} = \left(nS\frac{\lambda}{2} \right)\left(\frac{\pi}{\lambda} m_a q_{0z}^2 u \right) = N_{\lambda/2} J_a , \tag{28}$$

with $N_{\lambda/2}$ - the number of atoms in the volume $V = S\lambda/2$, $E_a = \left(m_a q_{0z}^2 \omega^2 \right)/2 = J_a \omega$ - the energy, $J_a = m_a q_{0z}^2 \omega/2 = \pi m_a q_{0z}^2 u/\lambda$ - the generalized momentum for microscopic/atomic oscillator given by the relations (17) and (22).

3. The Equivalent Mass of the Wave

3.1. The Mechanical Wave Mass

The macroscopic oscillator energy given by (25) can be expressed in terms of the velocity, considering the relation (5)

$$E_{\lambda/2} = \left(\frac{\pi}{2} \rho Skq_{0z}^2 \right) u^2 = \left(\pi^2 \rho S \frac{q_{0z}^2}{\lambda} \right) u^2 . \tag{29}$$

Introducing the notion of equivalent mass for a macroscopic oscillator corresponding to a half-wavelength

$$m_{u,\lambda/2} = \frac{\pi}{2} \rho Skq_{0z}^2 = 2\left(\rho S \frac{\lambda}{2} \right)\left(\frac{\pi q_{0z}}{\lambda} \right)^2 = \frac{m_{\lambda/2}}{2}\left(\frac{2\pi q_{0z}}{\lambda} \right)^2 , \tag{30}$$

with $m_{\lambda/2} = \rho S\left(\lambda/2 \right)$ the mass of macroscopic oscillator's substance (gas), we can write an Einstein type expression for energy (Feynman *et al*., 1964, Ch. 15.9)

$$E_{\lambda/2} = m_{u,\lambda/2} u^2 . \tag{31}$$

The linear momentum along the *Oz* direction corresponding to macroscopic oscillator, is

$$p_{z,\lambda/2} = m_{u,\lambda/2} u . \tag{32}$$

We can define also a linear momentum for wave, that is the root mean square of internal oscillation momentum (averaged over time and space) of atomic oscillators (usually, microscopic oscillators). From the momentum of an atomic oscillator

$$p_a = m_a \frac{\partial q_z}{\partial t} = m_a q_{0z} \omega \cos\left(\omega t - kz \right), \tag{33}$$

the root mean square atomic momentum is

$$\mathsf{P}_a = \sqrt{\left\langle p_a^2 \right\rangle_{z,t}} = \frac{1}{\sqrt{2}} m_a q_{0z} \omega . \tag{34}$$

The internal macroscopic momentum is the total momentum corresponding to the oscillators from a half-wavelength

$$P_{\lambda/2} = \left(nS\frac{\lambda}{2}\right)P_a = \sqrt{2}\left(\frac{\lambda}{2\pi q_{0z}}\right)m_{u\lambda/2}u = \sqrt{2}\left(\frac{\lambda}{2\pi q_{0z}}\right)p_{z\lambda/2} \tag{35a}$$

or

$$P_{\lambda/2} = \frac{1}{\sqrt{2}}\left(\frac{2\pi q_{0z}}{\lambda}\right)m_{\lambda/2}u_l \tag{35b}$$

and it is much higher than the wave momentum (32), for $\lambda >> q_{0z}$.

The ratio of the linear momentum of the oscillator (32) and action (26) is

$$\frac{p_{z,\lambda/2}}{J_{\lambda/2}} = k \tag{36}$$

a relationship between momentum and wave vector analogous to the photon (Bransden and Joachain, 1989, Ch. 1.3).

3.2. The Mass of Standing Wave

The j^{th} harmonics of standing wave in the gas-filled tube isa macroscopic oscillator corresponding to this tube is consisting of j half-wavelengthmacroscopic oscillators have, according to the relations (21) and (31), the energy

$$E_{\lambda_j/2} = \left(\frac{\pi}{4}\rho l_x l_y k_j q_{0szj}^2\right)u^2 = \left(\frac{\pi^2}{2}\rho l_x l_y \frac{q_{0szj}^2}{\lambda_j}\right)u^2 . \tag{37}$$

The equivalent mass, corresponding to this oscillator is

$$m_{us\lambda_j/2} = \frac{\pi^2}{2}\rho l_x l_y \frac{q_{0szj}^2}{\lambda_j} = \frac{m_{\lambda_j/2}}{4}\left(\frac{2\pi q_{0szj}}{\lambda_j}\right)^2 . \tag{38}$$

Linear momentum along *Oz* direction and the total mean momentum are null.

For the standing wave, we can define an internal linear momentum as root mean square momentum (averaged over time and space) of atomic oscillators (usually, microscopic oscillators). The momentum of an atomic oscillator in the standing wave is

$$p_{aj} = m_a\frac{\partial q_{szj}}{\partial t} = m_a q_{0szj}\omega\sin(k_j z)\cos(\omega_j t) . \tag{39}$$

Root mean square atomic momentum averaged in space and time is

$$P_{aj} = \sqrt{\left\langle p_{aj}^2\right\rangle_{z,t}} = \frac{1}{2}m_a q_{0szj}\omega_j . \tag{40}$$

The internal macroscopic momentum is the corresponding total momentum of oscillators from a half-wavelength

$$P_{\lambda_j/2} = \left(n\frac{\lambda_j}{2}l_x l_y\right)P_{aj} = \frac{1}{2}\left(\frac{\lambda_j}{\pi q_{0szj}}\right)\left(m_{us\lambda_j/2}u\right) = \left(\frac{2\pi q_{0szj}}{\lambda_j}\right)\left(m_{\lambda_j/2}u\right). \quad (41)$$

This momentum is different from expression (35) by the amplitude value ($q_{0szj} \neq q_{0z}$, for the standing wave with wavelength λ_j amplitudes are related by $q_{0szj} = 2q_{0zj}$) of the wavelength $\lambda_j \neq \lambda$.

4. The Inertial Mass of Wave

4.1. The Standing Wave Relative to an Inertial Reference System

For an observer moving with constant velocity υ , the standing wave ($\Psi_s\left(z,t\right) = q_{sz} = q_{0sz}\sin(kz)\sin(\omega t)$) is an amplitude modulated waveform (Elbaz, 1986; Kracklauer, 1999)

$$\Psi\left(z,t\right) = q_{sz} = q_{0sz}\sin\left[\gamma_u k_0\left(z-\upsilon t\right)\right]\sin\left[\gamma_u\omega_0\left(t-\frac{\upsilon}{u^2}z\right)\right]. \quad (42)$$

The wave is characterized by: angular frequency

$$\omega = \gamma_u\omega_0 > \omega_0 , \quad (43)$$

the wave vector

$$k = \gamma_u k_0\frac{\upsilon}{u} << k_0 \quad (43)$$

and the phase velocity

$$\upsilon_\varphi = \frac{\omega}{k} = \frac{u^2}{\upsilon} >> u . \quad (44)$$

The wave amplitude is modulated by: modulation frequency

$$\omega_{\mathrm{mod}} = \gamma_u k_0\upsilon = \omega_0\gamma_u\frac{\upsilon}{u} << \omega ; \quad (45)$$

the wave vector

$$k_{\mathrm{mod}} = \gamma_u k_0 \quad (46)$$

and group velocity equal to the speed of the observer

$$\upsilon_g = \frac{\omega_{\mathrm{mod}}}{k_{\mathrm{mod}}} = \upsilon << u . \quad (47)$$

The amplitude of oscillation is modulated ($q_{\mathrm{mod}} = q_{0sz}\sin\left[\gamma_u k_0\left(z-\upsilon t\right)\right]$) and the corresponding wavelength is

$$\lambda_{\mathrm{mod}} = \frac{2\pi}{k_{\mathrm{mod}}} = \frac{\lambda_0}{\gamma_u} < \lambda_0 . \quad (48)$$

The relationships above are obtained using Lorentz transformations and Doppler displacement relations, replace the mechanical speed of light c with the mechanical wave propagation velocity u.

4.2. The Waves Relative to a Non-Inertial Referential System

To deduce the physical significance of the equivalent mass for wave given by Eq. (29), we deduce the expression of wave energy and mass relative to anaccelerated system.

Consider an accelerated reference frame $S'(x', y', z', t')$ characterized by acceleration vector $\vec{a}(0, 0, a_z = a)$ relative to an inertial reference frame. The relations between z, t and proper time τ, relative to the reference frame instantly found the rest of the accelerated observer, Rindler observer (Rindler, 2006; Alsing and Milonni, 2004)

$$t(\tau) = \frac{u}{a}\sinh\left(\frac{a\tau}{u}\right), \quad z(\tau) = \frac{u^2}{a}\cosh\left(\frac{a\tau}{u}\right), \tag{49}$$

Relative to the reference frame being in instantly rest of the observer (the instantaneous rest frame of the observer) angular frequency ω' has the expression:

$$\omega'(\tau) = \frac{\omega - k\upsilon(\tau)}{\sqrt{1 - \upsilon^2(\tau)/u^2}} = \omega \exp\left(\frac{-a\tau}{u}\right), \quad k = \frac{\omega}{u}, \tag{50a}$$

for waves that propagate towards the observer acceleration

$$\omega'(\tau) = \omega \exp\left(\frac{a\tau}{u}\right), \quad k = -\frac{\omega}{u} \tag{50b}$$

and for waves that propagate in the opposite direction of acceleration observer.

To deduce the half-wavelength energy corresponding to a wave (Eq. (25)) relative to the Rindler observers consider that the generalized impulse is Lorentz invariant

$$J'_{\lambda/2} = J_{\lambda/2}. \tag{51}$$

In those conditions, the expression of energy (25) transform same as pulsation (50)

$$E'_{\lambda/2} = J'_{\lambda/2}\omega' = J_{\lambda/2}\omega'. \tag{52}$$

Substituting (50) in the expression (52) gives

$$E'_{\lambda/2} = J_{\lambda/2}\omega \exp\left(\frac{a\tau}{u}\right) = E_{\lambda/2} \exp\left(\frac{a\tau}{u}\right). \tag{53}$$

The corresponding power relative to Rindler observer is

$$\mathrm{P}'_{\lambda/2} = \frac{dE'_{\lambda/2}}{d\tau} = E_{\lambda/2}\left(\frac{\mp a}{u}\right)\exp\left(\frac{\mp a\tau}{u}\right) = F'_{\lambda/2}u. \tag{54}$$

From the Eq. (54), results

$$F'_{\lambda/2} = \frac{E_{\lambda/2}}{u^2}(\mp a)\exp\left(\frac{\mp a\tau}{u}\right). \quad (55)$$

Substituting in (55), the expression of equivalent mass of the wave, according to the Eq. (30), gives

$$F'_{\lambda/2} = (\mp a)m_{u,\lambda/2}\exp\left(\frac{\mp a\tau}{u}\right) = (\mp a)m'_{u,\lambda/2}. \quad (56)$$

From Eq. (56), results that the inertial mass relative to the Rindler observer is dependent on proper time τ

$$m'_{u,\lambda/2}(a,\tau) = m_{u,\lambda/2}\exp\left(\frac{\mp a\tau}{u}\right). \quad (57)$$

and $m_{u,\lambda/2} = m'_{u,\lambda/2}(a=0)$, the inertial mass of the wave relative to the un accelerated observer $S(x,y,z,t)$, is the equivalent mass of the wave. It is observed that the equivalent mass wave has the same transformation expressions as energy and angular frequency.

5. Conclusions

The energy carried by a mechanical wave depending on the speed of propagation highlights the existence of an equivalent mass of wave. The two quantities are linked by an Einstein type relation. This mass is analogous with the mass of electromagnetic wave. The mechanical wave has also a linear momentum and an action which are connected by an analogous relation to the momentum and the wavelength of photon.

The standing wave is characterized by an energy and an equivalent rest mass connected by an Einstein type relation.

Relative to a moving observer the standing wave is a wave that carries energy with transport speed of reference frame in the opposite direction of movement. The relations between the quantities of the standing wave and progressive wave are Lorentz type transformations, the speed of light is replaced with the mechanical wave propagation speed in the medium. The relationship between energy and equivalent mass of progressive wave is Einstein type. The action corresponding to the standing wave has the same expression as progressive wave, *i.e*. it is invariant under Lorentz type transformations. The progressive wave momentum defined as the equivalent mass multiplied by the propagation velocity has a de Broglie type relation with invariant action.

Studying the behavior of standing wave compared with an accelerated reference frame (Rindler transformations), demonstrate that the equivalent mass of the wave is an inertial mass (a measure of mechanical inertia of system).

A substantial medium where propagates mechanical disturbances is an *acoustic universe* and the maximum energy and information propagation speed is the mechanical waves speed in undisturbed environment. All the phenomena that take place in this *acoustics world* are causally correlated with the same speed. The coordinate and time transformations between two coordinate frames are Lorentz type transformation. The results of this paper are developed in the papers (Simaciu *et al*., 2015; Simaciu *et al*., 2016).

REFERENCES

Alsing P.M., Milonni P.W., *Simplified Derivation of the Hawking-Unruh Temperature for an Accelerated Observer in Vacuum*, American Journal of Physics, **72** 1524-1529 (2004).

Barceló C., Garay L.J., Jannes G., *Two Faces of Quantum Sound*, Phys. Rev. D., **82**, *4*, 044042 (2010).

Barceló C., Liberati S., Visser M., *Analogue Gravity*, Living Rev. Relativity, **14**, 3-160 (2011).

Bransden B.H., Joachain Ch.J., *Introduction to Quantum Mechanics*, Longman Scientific & Technical (1989).

Butikov E.I., *Peculiarities in the Energy Transfer by Waves on Strained Strings*, Phys. Scr. 88 065402 doi:10.1088/0031-8949/88/06/065402 (2013).

Elbaz C., *Propriete cinematiques des particules materielles et des ondes stationnaires du champ*, Annales de la Fondation Louis de Broglie, Vol. **11**, No. *1*, pp. 65-84 (1986).

Fasano A., Marmi St., *Analytical Mechanics*, English translation, Oxford University Press (2006).

Feynman Richard P., Leighton R.B., Sands M., *The Feynman Lectures on Physics*, Vol. **1**, Addison-Wesley, Reading (1964).

Kracklauer A.F., *Pilot Wave Steerage: A Mechanism and Test*, Foundations of Physics Letters, Vol. **12**, No. *2*, 441-453 (1999).

Landau L., Lifchitz E., *Mecanique des fluids*, Edition Mir, Moscou (1971).

Nandi K.K., Zhangb Yuan-Zhong, Caic Rong-Gen, *Acoustic Wormholes*, arXiv:gr-qc/0409085v5 18 Nov (2004); K.K. Nandi, D.H. Xu, *Unruh model for the Einstein-Rosen charge: Squealing Wormholes?*, arXiv:gr-qc/0410052v2 13 Oct (2004).

Rindler W., *Relativity-Special*, General and Cosmological, Second Edition, Oxford University Press Inc., New York (2006).

Simaciu I., Borsos Z., Dumitrescu Gh., Nan G., *Planck-Einstein-de Broglie Relations for Wave Packet: The Acoustic World*, arXiv:1511.01049 (2015).

Simaciu I., Dumitrescu Gh., Borsos Z., Brădac M., *Interactions in an Acoustic World*, arXiv:1612.00294 (2016).

Unruh W.G., *Sonic Analog of Black Holes and the Effects of High Frequencies on Black Holes Evaporation*, Phys. Rev. D, **51**, 2827-2838 (1995).

Visser M., *Acoustic Black Holes: Horizons, Ergospheres and Hawking Radiation*, Class. Quantum Grav., **15**, 1767-1791 (1998).

Weinfurtner S. *et al.*, *Classical Aspects of Hawking Radiation Verified in Analogue Gravity Experiment,* Lect. Notes Phys., **870**, 167-180 (2013); arXiv:1302.0375 [gr-qc].

LUMEA ACUSTICĂ: INERȚIA MECANICĂ A UNDELOR

(Rezumat)

În această lucrare analizăm fenomenele care se produc într-un mediu material în care energia și informația se propagă cu viteza undelor mecanice. Perturbațiile care se generează și evoluează/propagă în mediu se corelează prin intermediul undelor acustice (mecanice) care au viteza maximă. Mediu material și perturbațiile care se produc în el (unde, pachete de unde, bule, etc.) formează lumea acustică. Noi demonstrăm că unda are masă echivalentă. Masa echivalentă este legată de energia undei printr-o relație de tip Einstein. Evenimentele din lumea acustică sunt legate prin transformări de tip Lorentz. Studiind comportarea unei unde staționare în raport cu un observator accelerat (observatorul Rindler), demonstrăm că masa echivalentă a undei este masa inerțială. Pentru unda care se propagă, între impulsul liniar și impulsul generalizat (action variable) există o relație de tip de Broglie adică impulsul undei este proporțional cu numărul de undă al undei, coeficientul de proporționalitate fiind acțiunea.