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EMBEDDING THEORY IN COMPLEX SYSTEMS

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Abstract. In this article we present an introduction in the embedding theory, which is a mean to mathematically describe the very irregular physical processes occurring in complex systems, such as fluid turbulence or those processes occurred at the nanoscale.

Keywords: non-differentiable physical processes; Lagrangean physical systems.

1. Introduction

The modeling in physics is based on differential models that use ordinary differential equations and/or partial differential equations. The use of these equations does not allow, however, the modeling of sufficiently irregular dynamic behavior. In most of the real physical problems, some of the phenomena escape from modeling, either because we do not know them (and nothing tells us that those that escape have to be modeled by differentiable patterns), or because we do not know how to model them (*i.e.* the variation of the sun's flattening over time, which should be taken into account in the

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evolution studies of the solar system over long periods of time). Therefore, current physical patterns are *regular traces* of more complex dynamics, which are not directly accessible to us.

The embedding theory of dynamical systems consists of trying to model more general dynamics from which, the regular dynamics described by partial differential equations, derive.

2. Embedding Theory

The strategy of embedding theory was first outlined in (Cresson, 2003). In order to describe this strategy, a series of notions should be introduced. It is embedded a functional $L\left(x, dx/dt\right)$, defined for $x \in C^1(\mathbb{R})$ in $C^0(\mathbb{R})$ and an operator D that satisfies the following constants: i) D is defined on $C^0(\mathbb{R})$, ii) $D = d/dt$ on $C^1(\mathbb{R})$.

Noting with P the extended functional L, after the embedding, to $C^0(\mathbb{R})$ we have the following diagram:

$$\begin{array}{ccc} L\left(x, dx/dt\right) & \xrightarrow{LAP} & EL \\ \downarrow P & & \downarrow P \\ L\left(X, DX\right) & \xrightarrow{?} & P\left(EL\right) \end{array} \tag{1}$$

where *LAP* is the least action principle, (*EL*) is the Euler-Lagrange classical equation associated with L and (?) is, for the time being, an unknown principle of the minimum action, which have to be defined case by case. The ignorance results from the lack of a correct definition of extreme notion and variation for the extended function. The extreme will have to be searched so as to make the diagram (1) switchable. The study of the existence of such an extreme is called *the coherence theorem* in embedding theory. The central point remains the extension of the classical derivative to a more general functional space.

By this extension of the notion of derivative we can reach two distinct theories.

In one case, the initial *EL* equation is present in the extended *EL* equation, *P* (*EL*). The new derivative is reduced to the classical derivative when returning to classical processes. For example, in the case of stochastic embedding, the new operator reduces to the classical derivative when returning to differentiable deterministic processes (Cresson and Darses, 2007). The terminology used in this case is even embedding theory. The scheme used in this case is the following:

– we extend classical derivatives to a functional space $\boldsymbol{F}$ and defines an application $p : C^0 \rightarrow \boldsymbol{F}$ that associates each continuous function (differentiable or not) with a functional which has the meaning of extended derivative.

– we extend the ordinary differential equations or the partial differential equations using the functional as an extended derivative.

Therefore, the original equation will recover from the extended equation by restricting the functional space to $p\left(C^k\right)$, *k* depending on the order of the original equation. The typical example in this case is *fractional embedding* (Cresson, 2007).

In the second case, an additional parameter, *h*, is used, and the extended operator, D_h is reduced to the classical derivative only when. In this case, the original *EL* equation is not contained in the extended equation *P* (*EL*); however, we have a continuous deformation of the *P* (*EL*) equation, which depends on. In this case, the terminology of *deformation theory* is used. The scheme used in this case is the following:

– we define a family of functional $\{\boldsymbol{F}\}$ which depend of one or more parameters. For comprehension, consider a single parameter and therefore the functional family dependent on this parameter $\{\boldsymbol{F}\}_{p_0}$.

– we define an operator $\boldsymbol{D}_{p_0}$ in such a way as when $C^1 \subset \{\boldsymbol{F}\}_{p_0}$ we have $\boldsymbol{D}_{p_0}(x) = dx/dt$ for $x \in C^1$.

It is obvious that from a deformation we can hardly get information about the initial equation. Asymptotic solutions should be sought for $p_0 \to 0$.

An example of this is the *scale calculation*. In this case, it is desired to capture the type of regularity of the graph of a function (trajectories) starting from a family of approximations having the following behaviour: if $\varepsilon \neq 0$, then the approximation is a differentiable function, and if $\varepsilon = 0$, then we obtain the original, non-differentiable function. Here ε is the scale resolution. In this approach, the notion of minimal resolution appears to be necessary, but its definition still requires discussions, and there is no intrinsic definition of this notion because this is about choosing a constant, which in practice, would preserve the role of precision, what has physical meaning, but not mathematical one.

3. Conclusions

Linear or nonlinear Schrodinger's equation can be obtained as a result of a principle of the minimal action formulated in one of two cases (embedding or deformation). These results suggest the following.

i) Any natural equation with partial derivatives could be obtained as a result of a minimum action principle. As natural we understand partial differential equations well known in physics, such as, for example, the Navier-Stokes equations or Dirac equations.

ii) If things are as above, this suggests a deeper relationship between embedding theories and partial-differential equations. More specifically, embedding approaches suggest that modelling using partial-differential equations would only consider the regular part of dynamical behaviours. In the same time, these equations cannot surprise in any way the turbulent solutions. The existence of so-called *weak solutions* or *strong solutions* means the existence of a supplementary structure which exists beyond the regular solutions, and which can only be surprised by the principles of minimal non-differentiable action.

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TEORIA “EMBEDDING” PENTRU SISTEME COMPLEXE

(Rezumat)

În acest articol prezentăm o introducere în teoria “embedding”-ului, care este o metodă de descriere matematică a proceselor fizice foarte neregulate în sisteme complexe, cum ar fi turbulențele fluide sau acele procese care apar la nano-scală, folosind un operator de derivare extins.