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COHERENCE IN FRACTAL STRUCTURES

BY

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Abstract. We use the Scale Relativity Theory formalism in an arbitrary constant fractal dimension to show that for a two-dimensional non-differentiable and non-coherent fluid, for which we consider its entities as vortex-type objects, the coherence mechanism induces vortices streets. Moreover, if the fluid bears self-constraints from the two planes, the attractive or repulsive interaction force between the two planes can be determined. As a result, a Cazimir-type effect at small scales and a Tifft-type effect at large scales can appear. At nanoscale, these findings could explain the fractional or integer quantum Hall effect in graphenes.

Keywords: Scale Relativity Theory; structure coherence; Cazimir-type effect; Tifft-type effect; fractional or integer quantum Hall effect; graphenes.

1. Introduction

Nonlinearity manifests itself under many forms. One of these, the coherent structures, is of high interest. These structures can appear from small scales (nanoscale and mesoscopic scale) to large scales (infragalactic scale and extragalactic scale). For example, for small scale turbulence, the evidence of high-vorticity small-size filaments which were observed in Navier-Stokes

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equations simulations has provided significant theoretical and experimental data (Kawahara and Kida, 2004; Reguera *et al*., 2008). Moreover, pattern formation and spatio-temporal structures are also prominent in fluid dynamics, dendritic growth, and alos chemo-biological phenomena. In adition, granular flow and fracture dynamics are new theoretical fields, which had given rise to numerous problems with important nonlinear and statistical aspects, and they will certainly be of great importane in the coming years (Reguera *et al.*, 2008). These same aspects can also be encountered at large scale in the forming processes of cosmic structures (Kauffmann *et al*., 1993; Schive *et al*., 2014).

The role coherence plays in structure formation at various scales is presented in (Gottlieb *et al*., 2004; Munceleanu *et al*., 2011; Timofte *et al*., 2011). More recently, the same topic has been discussed in various models of biological systems in (Tesloianu, 2015; Tesloianu *et al*., 2015), and particularly for blood assimilated to a complex fluid.

In this work we want to show that in the case of a complex fluid, no matter the scale, coherence induces interaction between the complex fluids' structural units.

2. Short Reminder on the Differentiable-Non-Differentiable Scale Transition Equations

The dynamics of the differentiable-non-differentiable scale transition at nanoscale are described as follows (Agop and Casian-Botez, 2015):

i) the specific momentum conservation law associated to differentiable-non-differentiable scale transition:

$$\partial_t \mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{V} - D(dt)^{\left(2/D_F\right)-1}\Delta\mathbf{V} = 2\mathbf{V}_F\cdot\nabla\mathbf{V}_F + 2D(dt)^{\left(2/D_F\right)-1}\nabla\mathbf{V}_F \quad (1)$$

ii) the states density conservation law associated to differentiable-non-differentiable scale transition:

$$\partial_t \rho + (\mathbf{V}\cdot\nabla)\rho - D(dt)^{\left(2/D_F\right)-1}\Delta\rho = 0 \quad (2)$$

In relations (1) and (2) $\mathbf{V}$ is the velocity associated to differentiable-non-differentiable scale transition

$$\mathbf{V} = \mathbf{V}_D - \mathbf{V}_F \quad (3)$$

$\mathbf{V}_D$ is the differentiable and scale independent velocity, $\mathbf{V}_F$ is the non-differentiable and scale dependent velocity (Nottale, 1993; Nottale, 2011), $\mathbf{V}\cdot\nabla\mathbf{V}$ is the convective-type term, $D(dt)^{\left(2/D_F\right)-1}\Delta\mathbf{V}$ is the dissipative-type term, D_F is the fractal dimension of the motion curves, dt is the scale resolution and D is the specific coefficient associated to the differentiable-non-differentiable scale transition. For D_F we can accept any definition (Kolmogorov fractal dimension, Hausdorff-Beskovici fractal dimension (Mandelbrot, 1983) etc.), but once a definition is set, it has to be constant over the entire theoretical model for the involved dynamics.

If the motions at non-differentiable scale are irrotational, *i.e.* $\nabla \times \mathbf{V}_F = 0$ we can choose $\mathbf{V}_F$ of the form

$$\mathbf{V}_F = D(dt)^{\left(2/D_F\right)-1} \nabla \ln \phi \tag{4}$$

with $\ln \phi$ the non-differentiable velocity scalar potential.

In the particular case $\phi \equiv \rho$ the right-side term from Eq. (1),

$$\begin{gathered} 2\left[\mathbf{V}_F \cdot \nabla \mathbf{V}_F + D(dt)^{\left(2/D_F\right)-1} \Delta \mathbf{V}_F\right] \equiv \\ \equiv -2\nabla\left[-\frac{\mathbf{V}_F^2}{2} - D(dt)^{\left(2/D_F\right)-1} \nabla \cdot \mathbf{V}_F\right] \equiv -\nabla \bar{Q} \end{gathered} \tag{5}$$

where $\bar{Q}$ is the specific non-differentiable potential associated to the differentiable-non-differentiable scale transition,

$$\bar{Q} \equiv -\mathbf{V}_F^2 - 2D(dt)^{\left(2/D_F\right)-1} \nabla \cdot \mathbf{V}_F \tag{6}$$

can be correlated with the tensor

$$\tau_{\mu\nu} \equiv 4D^2 (dt)^{\left(4/D_F\right)-2} \rho \nabla_\mu \nabla_\nu \ln \rho \tag{7}$$

by means of relation

$$\nabla \hat{\tau} + \rho \nabla \bar{Q} \equiv 0 \tag{8}$$

For „fluid" behaviours at differentiable-non-differentiable scale transition of isentropic type Eq. (7) becomes (Lifshiëtìs and Landau, 1987)

$$\tau_{\mu\nu} = -p\delta_{\mu\nu} \tag{9}$$

where p is the pressure and

$$\delta_{\mu\nu} = \begin{cases} 1 & \mu = \nu \\ 0 & \mu \neq \nu \end{cases} \tag{10}$$

Next, we want to demonstrate that the above-defined pressure can generate either atractive, or repulsive force fields. In order to acomplish wemust firstly consider that the velocity field is a cnoidal-type one (for mode details on the subject, see (Casian-Botez and Agop, 2015)).

3. Chaoticisation Through Non-Differentiability

All physical variables cuantities, which are dependent on spatial-temporal coordinates and resolution scales (*i.e.* fractal variables), can be extended on a complex manifold by means of chaoticisation through non-differentiability (Nottale, 1993; Nottale, 2011). As an example, in the case of real space, the scalar velocity potential can be replaced with a "state function" from the fractal space (with probabilistic meanings of state density) through such an extension. Thus, the "state function's" form can be determined through self-similarity that characterizes fractal variables (Aronstein and Stround, 1997; Cristescu, 2008): if, in the real space, the one-dimensional velocity is of a cnoidal type (more details on this subject can be found in (Casian-Botez and Agop, 2015)), then, in the fractal space, the "state function" will also be cnoidal, if we use a suitable selection of a normalization factor.

Let us now consider a two-dimensional non-differentiable and non-coherent fluid. Then its entities, assimilated to vortex-type objects, are structured as a two-dimensional lattice, as can be seen in Fig. 1.

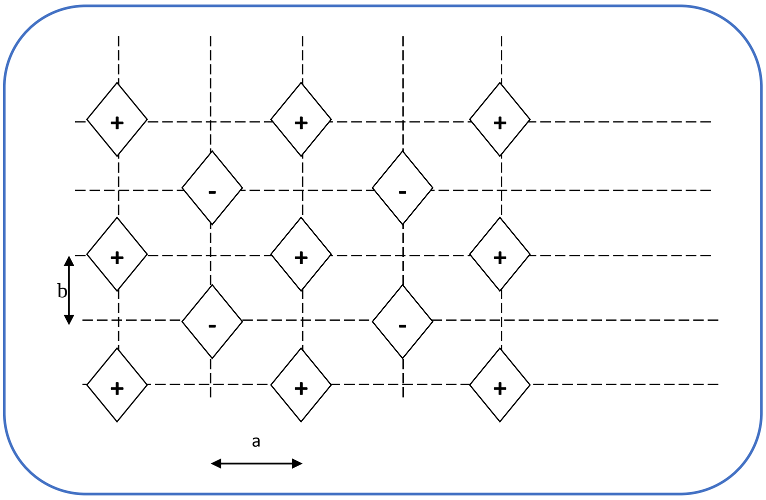

Fig. 1 – A two-dimensional lattice of vortex-type objects.

Then, taking into consideration the facts presented above (the cnoidal mode which is assimilated to a Toda-type nonlinear lattice (Cristescu, 2008; Toda, 1989) and the self-similarity property of physical variables) the "state function" has the expression

$$\Psi = cn\left(\bar{v}, s\right) \tag{11}$$

with

$$\begin{aligned}
&\bar{v} = \frac{K}{a}\bar{u}, \bar{u} = \xi + i\eta, \frac{K}{a} = \frac{K^{'}}{b},\\
&K = \int_{0}^{\frac{\pi}{2}} \frac{d\varphi}{\left(1-\bar{k}^{2}\sin^{2}\varphi\right)^{\frac{1}{2}}}, K^{'} = \int_{0}^{\frac{\pi}{2}} \frac{d\varphi}{\left(1-\bar{k^{'}}^{2}\sin^{2}\varphi\right)^{\frac{1}{2}}},\\
&\overline{k^{2}} + \overline{k^{'2}} = 1
\end{aligned} \tag{12a-f}$$

In relations (12 a-f) K, $K^{'}$ are the complete elliptic integrals of the first kind of modulus $\bar{k}$ [37] and *a*, *b* are the constants of the vortex lattice (Armitage and Eberlein, 2006).

If we apply this formalism to a complex plane (for details see (Lifshiëtìs and Landau, 1987)) and using the following equation

$$\Psi = e^{Q(\bar{u})/\Gamma} \equiv cn\left(\bar{v};\bar{k}\right) \tag{13}$$

we induce the scalar complex potential of the complex velocity field

$$Q\left(\bar{u}\right) = \Gamma \ln\left[\mathrm{cn}\left(\bar{v};\bar{k}\right)\right] \tag{14}$$

with Γ the vortex constant.

Based on (14) the complex velocity field can then be defined as

$$V_\xi - iV_\eta = \frac{dQ(\bar{u})}{d\bar{u}} = -\frac{\Gamma K}{a}\frac{\mathrm{sn}(\bar{v};\bar{k})\,dn(\bar{v};\bar{k})}{\mathrm{cn}(\bar{v};\bar{k})} \tag{15}$$

or, using the notations (Armitage and Eberlein, 2006)

$$\begin{aligned} &\bar{s} = \mathrm{sn}(\bar{\alpha};\bar{k}), \bar{c} = \mathrm{cn}(\bar{\alpha};\bar{k}), \bar{d} = \mathrm{dn}(\bar{\alpha};\bar{k}),\\ &\bar{\alpha} = \frac{K}{a}\xi, \overline{s_1} = \mathrm{sn}\left(\bar{\beta},\overline{k^{'}}\right),\\ &\overline{c_1} = cn\left(\bar{\beta},\overline{k^{'}}\right), \overline{d_1} = dn\left(\bar{\beta},\overline{k^{'}}\right), \bar{\beta} = \frac{K}{a}\eta \end{aligned} \tag{16a-h}$$

$$\begin{aligned} V_\xi - iV_\eta = &-\frac{\Gamma K}{a}\frac{\overline{scd}\left[\overline{c_1}^2\left(\overline{d_1}^2 + \bar{k}^2\bar{c}^2\overline{s_1}^2\right) - \overline{s_1}^2\overline{d_1}^2\left(\bar{d}^2\overline{c_1}^2 - \bar{k}^2\bar{s}^2\right)\right]}{\overline{scd}\left[\overline{c_1}^2\left(\overline{d_1}^2 + \bar{k}^2\bar{c}^2\overline{s_1}^2\right) - \overline{s_1}^2\overline{d_1}^2\left(\bar{d}^2\overline{c_1}^2 - \bar{k}^2\bar{s}^2\right)\right]} -\\ &-i\frac{\Gamma K}{a}\frac{\overline{s_1}\,\overline{c_1}\,\overline{d_1}\left[\bar{c}^2\left(\bar{d}^2\overline{c_1}^2 - \bar{k}^2\bar{s}^2\right) + \bar{s}^2\bar{d}^2\left(\overline{d_1}^2 + \bar{k}^2\bar{c}^2\overline{s_1}^2\right)\right]}{\left(1-\bar{d}^2\overline{s_1}^2\right)\left(\bar{c}^2\overline{c_1}^2 + \bar{s}^2\bar{d}^2\overline{s_1}^2\overline{d_1}^2\right)} \end{aligned} \tag{17}$$

Since

$$\begin{aligned} &\mathrm{cn}(\bar{v} + \bar{\Omega}) = \mathrm{cn}(\bar{v})\\ &\bar{\Omega} = 2(2m+1)K + 2inK^{'}\\ &m, n = \pm 1, \pm 2, \ldots \end{aligned} \tag{18a-c}$$

for $\bar{k} \to 0, \overline{k^{'}} \to 1$ and $\bar{k} \to 1, \overline{k^{'}} \to 0$ limits, the initially non-coherent fluid (with the amplitudes and phases of its entities independent) becomes coherent (i.e. the amplitudes and phases of its entities are starting to be correlated). These types of dynamics can be seen in Figs. 2 *a-f*: it results that the coherence of the fluid reduces to its ordering on vortices streets – see Figs. 2 *a, b* for vortices streets aligned with the $O\xi$ axis and Figs. 2 *e, f* for vortices streets aligned with the $O\eta$ axis.

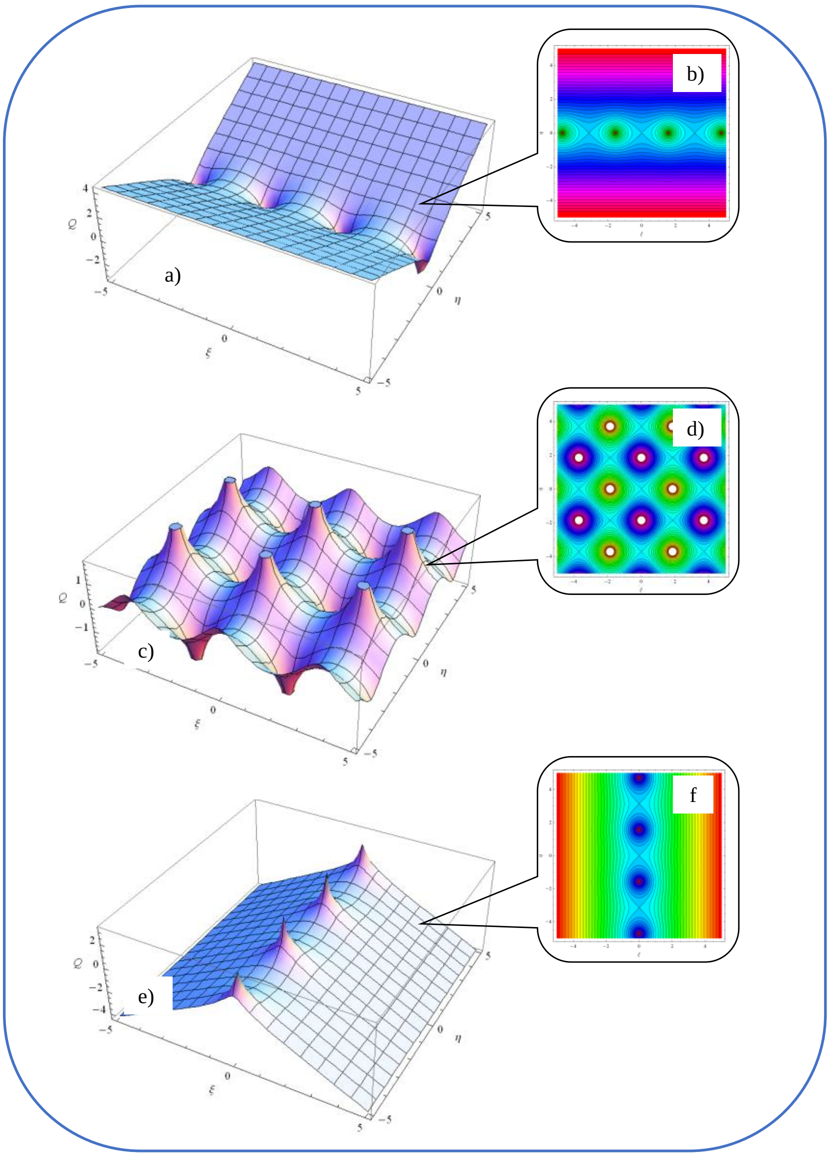

Fig. 2 – Three–dimensional (a, c, e) and two-dimensional (b, d, f) real part of the potential velocity field for different nonlinearity degrees ($s = 0.1$ – a, b; $s = 0.5$ – c, d; $s = 1$ – e, f).

In this manner, if we consider that the state density is constant, the difference between self-dissipation and self-convection generates, through a self-pressure gradient, the self-force:

$$\frac{1}{\rho}\nabla p = \Gamma\Delta \boldsymbol{V} - \boldsymbol{V}\cdot\Delta \boldsymbol{V} \tag{19}$$

or, in the ξ, η coordinates plane

$$\begin{aligned}\frac{\partial p}{\partial \xi} &= \rho\Gamma\left(\frac{\partial^2 V_\xi}{\partial \xi^2}+\frac{\partial^2 V_\xi}{\partial \eta^2}\right)-\rho\left(V_\xi\frac{\partial V_\xi}{\partial \xi}+V_\eta\frac{\partial V_\xi}{\partial \eta}\right)\\ \frac{\partial p}{\partial \eta} &= \rho\Gamma\left(\frac{\partial^2 V_\eta}{\partial \xi^2}+\frac{\partial^2 V_\eta}{\partial \eta^2}\right)-\rho\left(V_\xi\frac{\partial V_\eta}{\partial \xi}+V_\eta\frac{\partial V_\eta}{\partial \eta}\right)\end{aligned} \tag{20a, b}$$

Then, after employing a quite long but elementary calculus one gets from (20a,b), through the degenerations:

i) $\bar{k}=0, \overline{k^{'}}=1, K=\frac{\pi}{2}, K^{'}=\infty$

$$\begin{aligned}p_\eta\left(\overline{\alpha_1}\right) &= -p_0\sin h^2\left(\frac{\pi l_1}{2a}\right)\frac{1-\tan^2\overline{\alpha_1}}{\cos\left(2\overline{\alpha_1}\right)+\cos h\left(\frac{\pi l_1}{2a}\right)}\\ p_\xi\left(\overline{\beta_1}\right) &= -p_0\sin^2\left(\frac{\pi l_2}{2a}\right)\frac{1-tan h^2\overline{\beta_1}}{\cos\left(\frac{\pi l_2}{2a}\right)+\cos h\left(2\overline{\beta_1}\right)}\end{aligned} \tag{21a, b}$$

with

$$p_0=\rho\left(\frac{\pi\Gamma}{a}\right)^2, \overline{\alpha_1}=\frac{\pi\xi}{2a}, \overline{\beta_1}=\frac{\pi\eta}{2a} \tag{22a-c}$$

ii) $\bar{k}=1, \overline{k^{'}}=0, K=\infty, K^{'}=\frac{\pi}{2}$

$$\begin{aligned}p_\eta\left(\overline{\alpha_1^{'}}\right) &= -p_0^{'}sin^2\left(\frac{\pi l_1}{2b}\right)\frac{1+\tan h^2\overline{\alpha_1^{'}}}{\cos\left(\frac{\pi l_1}{b}\right)+\cos h\left(2\overline{\alpha_1^{'}}\right)}\\ p_\xi\left(\overline{\beta_1^{'}}\right) &= -p_0^{'}sinh^2\left(\frac{\pi l_2}{2b}\right)\frac{1-tan^2\overline{\beta_1^{'}}}{\cos\left(2\overline{\beta_1^{'}}\right)+\cos h\left(\frac{\pi l_2}{2b}\right)}\end{aligned} \tag{23a, b}$$

with

$$p_0^{'}=\rho\left(\frac{\pi\Gamma}{2b}\right)^2, \overline{\alpha_1^{'}}=\frac{\pi\xi}{2b}, \overline{\beta_1^{'}}=\frac{\pi\eta}{2b} \tag{24a-c}$$

In relations (21a, b) – (24a-c) l_1 and l_2 are the elementary space intervals as considered on the $O\eta$ and $O\xi$ axis, respectively (Fig. 3). As a result, we can state that the non-differentiability and coherence properties of the fluid, due to self-constraints, generate pressure along the $O\xi$ and $O\eta$ axis.

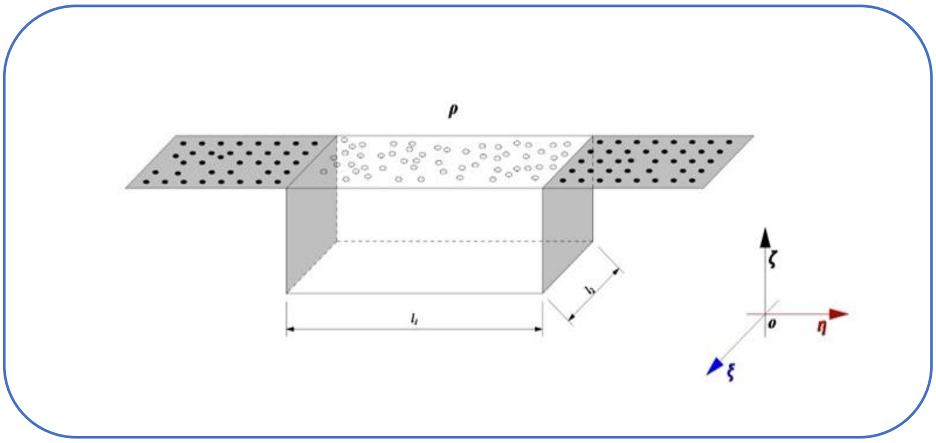

Fig. 3 – The fluid between two parallel planes, with its entities assimilated to vortex – type objects.

Let us now envision a fluid with a vortex lattice bounded by two parallel and infinitely thin liquid planes in the $\xi O\zeta$ plane, at a distance l_1 of each other. According to the facts we presented, if the fluid bears self-constraints from these two planes, then on their normal axis (here, $O\eta$ axis), a coherent structure of vortex street type is induced. Consequently, by integrating (23a, b) and (24a-c) in relation with variables $\overline{\alpha_r}$ and $\overline{\beta_r}$ and under restrictions

$$\begin{aligned} &l_1 \approx \delta\pi b, \\ &l_2 \approx \nu\pi a, \\ &\nu,\ \delta\ =\ 1, 2, \ldots \end{aligned} \tag{25a-c}$$

This is shown in Figs. 4*a*, *b* (for different values of the parameters ν, $\delta = 1, 2, \ldots$ and r).

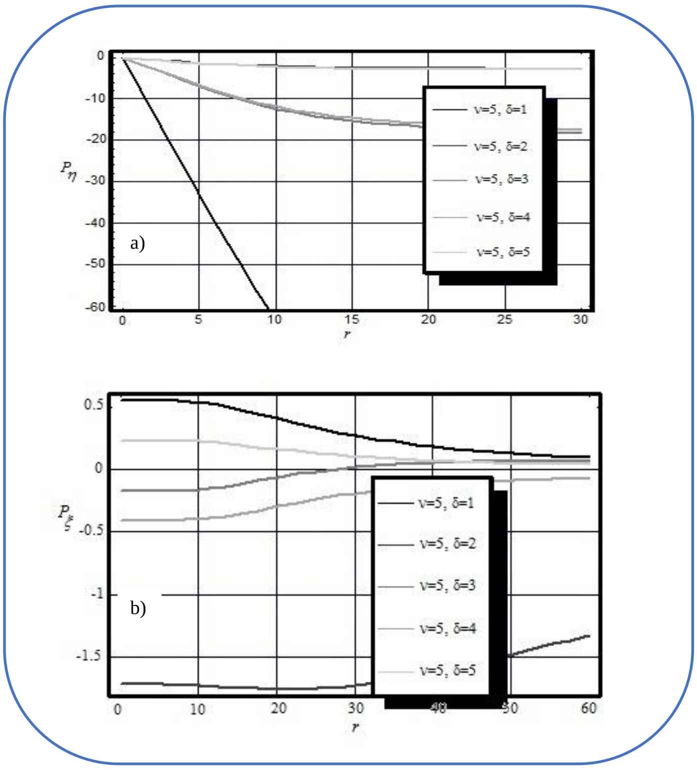

Fig. 4 – Plot of pressure p_η on the planes, versus parameter r for $\nu = 5$, $\delta = 1,..5$ (*a*); Plot of pressure p_ξ versus parameter r for $\nu = 5$, $\delta = 1,..5$ (*b*).

We must highlight the following conclusions: a) pressure p_η on the planes, given by (26a) stabilized for great r values, is always negative, hence an attractive force (Fig. 4*a*); b) besides pressure p_η acting on the planes, another pressure must manifest, p_ξ (Fig. 4*b*), acting along the $O\xi$ axis and given by (26b). Thus we notice that this pressure becomes null for great r values, and has a minimum for some values of the parameters m, n; c) if the planes were in the $\eta O\zeta$ plane, the self-constraints being along the $O\xi$ axis, vortices streets would form along this axis and the result in (23a, b) with (24a-c) would have been applied, *i.e.* the cases i) or ii) are identical, nonetheless they depend on the selected geometry; d) the pressures $p_\xi\left(\overline{\beta_1}\right)$ and $p_\xi\left(\overline{\beta'_1}\right)$ generate tensions of internal friction, while $p_\eta\left(\overline{\alpha_1}\right)$ and $p_\eta\left(\overline{\alpha'_1}\right)$ generate compression tensions in the attractive case and stretching tensions in the repulsive case; e) if one tries to compute the order of magnitude of the force between the planes, and replaces in (23b) or (25b): $\Gamma = h/2\pi m = 1.054 \cdot 10^{-34}\,Js/10^{-27}kg \approx 10^{-7}m^2/s$,

$\rho \approx 10^3 kg/m^3$ $(a, b) \approx 10^{-6}m$ (specific values for the boundary layer) and $l_1, l_2 \approx 5a, 5b$ (the distances between the planes) a value for $p_\xi\left(\overline{\beta_1}\right)$ can be obtained, $p_\xi\left(\overline{\beta'_1}\right) \approx 10N/m^2$, *i.e.* of the order of viscous dissipation tension (Lifshiëtìs and Landau, 1987). A similar calculus can be made for Cooper-type pairs in the case of type I superconductors (Poole *et al*., 1995).

4. Conclusions

The main conclusions of the present paper are presented in the following:

i) A short description of the differentiable-non-differentiable scale transition dynamics is made (implying momentum and states density conservation laws).

ii) Applying this specific formalism, it can be shown that, in the case of a two-dimensional non-differentiable and non-coherent fluid, with its entities assimilated to vortex-type objects, the coherence induces vortices streets.

iii) Furthermore, if the fluid bears self-constraints from the two planes, then on their normal axis a coherent structure of vortex street type appears. In this case, the interaction forces (being either attractive or repulsive) between the two planes can be assessed. Then, a Cazimir-type effect (Wilson *et al*., 2011) at small scales and a Tifft-type effect (Tifft, 1982) at large scales can manifest. At nanoscales, such an effect could explain the fractional or integer quantum Hall effect (Rao and Sood, 2013) in graphenes.

iv) This theoretical model can be applied to infra and extra galactic scales, for which the vortex constant is related to a gravitational-type Planck constant (Agnese and Festa, 1997).

v) Moreover, in our opinion, by being able to understand the rules which determine the structure coherence of complex fluids, one cand find the most viable solution for explaining the specific individual variations in the evolution and prognosis of different types of cardiovascular diseases (Mäkikallio *et al*., 2001).

We note that the same model can also be applied, because of its theoretical implications, in engineering and materials science, in various domains, such as the ones described in (Agape *et al*., 2016; Agape *et al*., 2017; Gaiginschi and Agape, 2016; Gaiginschi *et al*., 2011; Gaiginschi *et al*., 2014a; Gaiginschi *et al*., 2014b; Gaiginschi *et al*., 2017; Vornicu *et al*., 2017).

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COERENŢA ÎN STRUCTURILE FRACTALE

(Rezumat)

Prin aplicarea Teoriei Relativităţii de Scară într-o dimensiune fractală de constantă arbitrară, se arată că, pentru un fluid necoerent nediferenţiabil bidimensional, ale cărui entităţi pot fi asimilate cu obiecte de tip vortex, mecanismul de coerenţă induce străzi de vortexuri. Într-un caz particular, dacă fluidul prezintă limitări date de cele două plane, forţa de interacţune (fie de tip atractiv, fie de tip repulsiv) dintre cele două plane poate fi determinată. Atunci, se pot observa efecte de tip Cazimir la scări mici şi efecte de tip Tifft la scări mari (extragalactice). La nanoscară, acestea pot explica efectul Hall fracţionar sau integru în grafene.