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INTEGER AND FRACTIONAL HALL-TYPE EFFECTS INDUCED BY FRACTALITY

BY

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Abstract. In the framework of the Scale Relativity Theory in an arbitrary and constant fractal dimension, some problems about the integer and fractional quantum Hall effects are analyzed. Thus, if the motions of the complex system entities take place on Peano curves, at Compton scale, in strong magnetic fields, the integer quantum Hall effect is obtained, while for the motions of the same entities which take place on other non-differentiable curves with their fractal dimensions different than 2, the fractional quantum Hall effect results.

Keywords: fractal models; integer and fractional Hall effect; Scale Relativity Theory.

1. Introduction

The integer and fractional quantum Hall effects manifest in two-dimensional electron systems at high magnetic fields and are usually defined by the origin of the underlying energy gaps in these two cases (Prange and Girvin, 1987). The integer quantum Hall effect can be explained by the energy gaps created by the spin-split Landau levels, which are discrete and highly degenerative. These levels correspond to the resolution of the single-particle

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energy spectrum induced by the magnetic field. In this case, each Hall plateau is in relation to the complete filling of an integer number of these single-particle levels. On the other hand, the fractional effect occurs only at certain partial fillings of the Landau levels. For this effect, the requisite energy gap comes entirely from multiple-bodies effects. These differences, although useful, can often be misleading, especially for the case of the odd-integer quantum Hall effect. For these states, in which the Fermi level resides in the spin-flip gap in the top Landau level, it has been shown that electron-electron interaction effects greatly enhance the energy gap above the single-particle Zeeman energy (Usher *et al*., 1990). The current theory is that the odd-integer quantum Hall effect states would survive even if the Zeeman energy was removed entirely (Sondhi *et al*., 1993). If this was the case, the two-dimensional electron systems could develop a spontaneous ferromagnetic order (at zero temperature) as a result of interaction effects. Recently, prediction has been made concerning the nature of the elementary excitations of these ferromagnetic states (Fertig *et al*., 1994). Supposing that the Zeeman energy is sufficiently small, the lowest-lying charged excitation is not simply a single flipped spin but a large, smooth distortion of the spin field for which numerous spins are flipped. These types of excitation, with charge $\pm e$ cost more Zeeman energy than a single spin flip; instead, the near-parallelism of neighboring spins saves on exchange energy. The total spin, and thus the spatial extent of these objects is determined by the competition which occurs between these two energies. Such unusual excitations (also known as “Skyrmions” in the limit of zero Zeeman energy) have been uncovered recently in nuclear magnetic resonance studies which involved ground state spin polarization of two-dimensional electron systems (Barrett *et al*., 1995). Recent works have uncovered that while large-spin Skyrmionic quasiparticles dominate the integer quantum Hall effects (where only the lower spin branch of the lowest Landau level is occupied), they are not relevant to the higher odd-integer states (Schmeller *et al*., 1995). In the present paper, using the motion fractal theory in the form of the Scale Relativity Theory in an arbitrary and constant fractal dimension, some problems about the integer and fractional quantum Hall effects are analyzed

2. Mathematical Model

Let us consider that the entities of a complex system move on continuous but non-differentiable curves (fractal curves). In this context, using a fractal theory of motion in the form of the Scale Relativity Theory in an arbitrary and constant fractal dimension (Nottale, 2011; Mercheș and Agop, 2016) the velocity becomes a complex variable in the form:

$$\hat{V}^i = -2i\lambda\left(dt\right)^{\left(2/D_F\right)-1}\partial^i \ln\psi \tag{1}$$

where ψ is the state function, λ is a coefficient associated to the fractal-non-fractal transition, *dt* is the scale resolution and D_F is the fractal dimension of the motion curve.

If $\psi = \sqrt{\rho} e^{iS}$, where $\sqrt{\rho}$ is an amplitude and *S* a phase, the complex velocity (1) has both a real component (at a differentiable scale resolution)

$$V^i = 2\lambda \left(dt\right)^{\left(2/D_F\right)-1} \partial^i S \tag{2}$$

and an imaginary component (at a fractal scale resolution)

$$U^i = \lambda \left(dt\right)^{\left(2/D_F\right)} \partial^i \ln \rho \tag{3}$$

In such a context, the coherence of the fractal fluid entities (Nottale, 2011; Mercheș and Agop, 2016), by applying the magnetic field $\mathbf{B} = \mathrm{rot}\mathbf{A}$, with $\mathbf{A}$ the vector potential of this field, imply through relation

$$\oint m_0 \mathbf{V}.d\mathbf{r} = q\oint \mathbf{A}.d\mathbf{r} = q\iint \mathrm{rot}\mathbf{A}.d\mathbf{\Sigma} = q\Phi \tag{4}$$

the quantification of the fractal magnetic field Φ as:

$$\begin{gathered} 2m_0\lambda\left(dt\right)^{\left(2/D_F\right)-1}\oint \nabla S d\mathbf{r} = 4\pi m_0 n\lambda\left(dt\right)^{\left(2/D_F\right)-1} = q\Phi \\ n = 1, 2, \ldots \end{gathered} \tag{5}$$

or more

$$\Phi = n\Phi_0 \tag{6}$$

where we will call the quantity

$$\Phi_0 = \frac{4\pi m_0}{q}\lambda\left(dt\right)^{\left(2/D_F\right)-1} \tag{7}$$

a fractal fluxon. In relations (4) and (5) m_0 is the mass of the fractal fluid entity, q is its electric charge (usually $q \equiv e$) and $d\mathbf{\Sigma}$ the elementary vector surface. By employing now mathematical procedures which are similar to the ones in (Peitgen *et al*., 2004; Stana *et al*., 2009), for motions on Peano curves ($D_F = 2$) at Compton scale ($\lambda = \hbar/2m_0$) for the fractal fluid entities (quantum fluid in our case) the Hall conductivity can be defined as:

$$G_H = \upsilon \frac{e^2}{h} \tag{8}$$

where υ is the filling factor. Thus, we can explain an integer quantum Hall effect. If the motions of the fractal fluid entities are different than the ones on Peano curves, but take place at the same Compton scale, then the Hall conductivity can be defined as:

$$G_{HF} = \mu \frac{e^2}{h} \tag{9}$$

where the filling factor $\mu \sim (dt)^{\left(2/D_F\right)-1}$ can be identified with continuous fractions, as specified in (Peitgen *et al*., 2004; Lakhtakia *et al*., 1988). Thus, a fractional quantum Hall effect is explained (for details see (Haldane, 1983; Halperin, 1984; Peña, 2014; Chien, 2013)).

3. Conclusions

In the present paper, some theoretical aspects on the integer and fractional Hall effects are presented. Thus, if the motions of the complex system entities take place on Peano curves, at Compton scale, in strong magnetic fields, the integer quantum Hall effect is obtained. For the motions of the same entities which take place on other non-differentiable curves with their fractal dimensions different than 2, the fractional quantum Hall effect results. These results can be applied to various technological fields, such as nanorobotics and nanomaterials engineering (Agape *et al*., 2016; Agape *et al*., 2017; Gaiginschi and Agape, 2016; Gaiginschi *et al*., 2011; Gaiginschi *et al*., 2014a; Gaiginschi *et al*., 2014b; Gaiginschi *et al*., 2017; Vornicu et al., 2017).

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EFECTE DE TIP HALL ÎNTREG ȘI FRACȚIONAR INDUSE DE FRACTALITATE

(Rezumat)

Utilizând modelul fractal al mișcării sub forma Teoriei Relativității de Scală în dimensiune fractală arbitrară și constantă, sunt analizate câteva aspecte teoretice asupra efectului Hall cuantic întreg și fracționar. Astfel, dacă mișcările entităților unui sistem complex situate într-un câmp magnetic puternic au loc pe curbe fractale Peano la scală Compton, atunci vom obține expresia conductanței în efect Hall cuantic întreg. Dinpotrivă, dacă mișcările entităților aceluiași sistem au loc pe curbe fractale în dimensiune fractală diferită de 2, atunci se obține expresia conductanței corespunzătoare efectului Hall cuantic fracționar.